Image Denoising Algorithm with a Three-Dimensional Grid System of Coupled Nonlinear Elements

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1. Abstract

This poster presents an image denoising algorithm with a three-dimensional grid system of time-evolving nonlinear elements. At each grid point, a nonlinear element is placed and coupled with its first nearest neighboring elements. We confirm performance and convergence of the algorithm on artificial and real images in comparison with two classical algorithms of a diffusion equation and median filtering.

2. Nonlinear Element[1,2]

$$\frac{du}{dt} = f(u,v) = \frac{1}{\epsilon}[u(u-a)(1-u)-v]$$

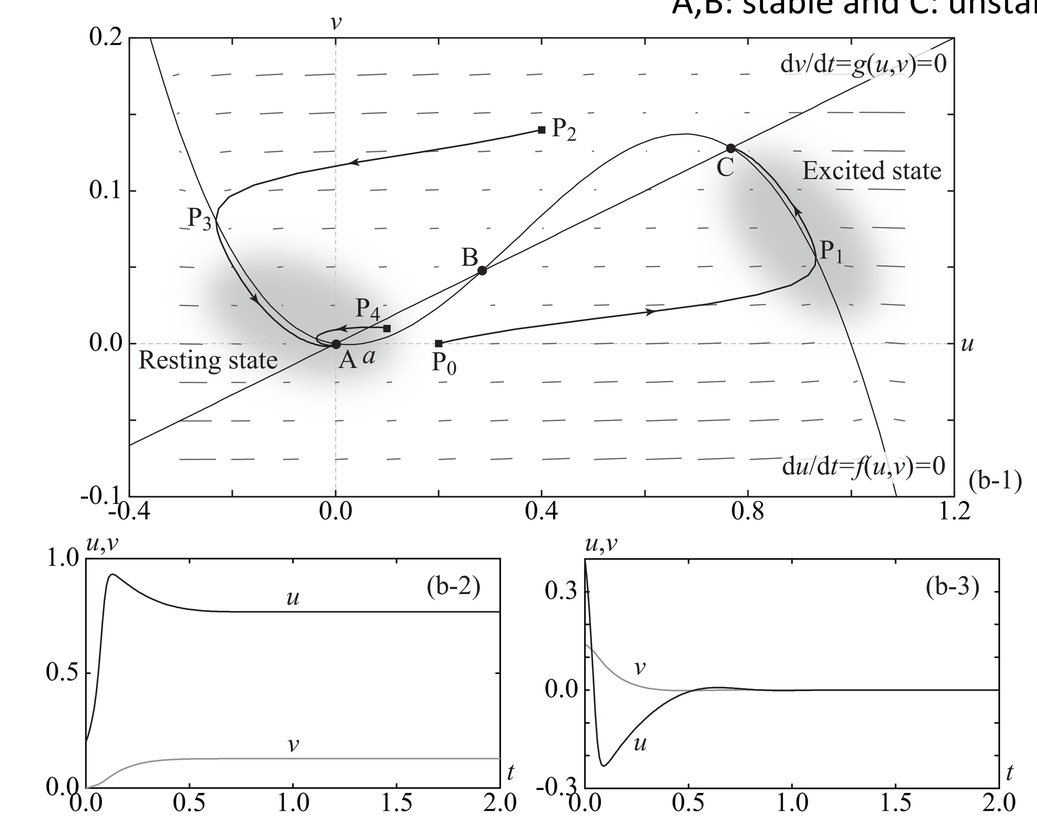
$$\frac{dv}{dt} = g(u,v) = u - bv$$

$u(t), v(t)$: variables defined at time $t > 0$.
$f(u,v), g(u,v)$: reaction functions
a, b, ϵ: constants

The nonlinear element behaves excitable or oscillatory, depending on its parameter settings (see the next phase plot).

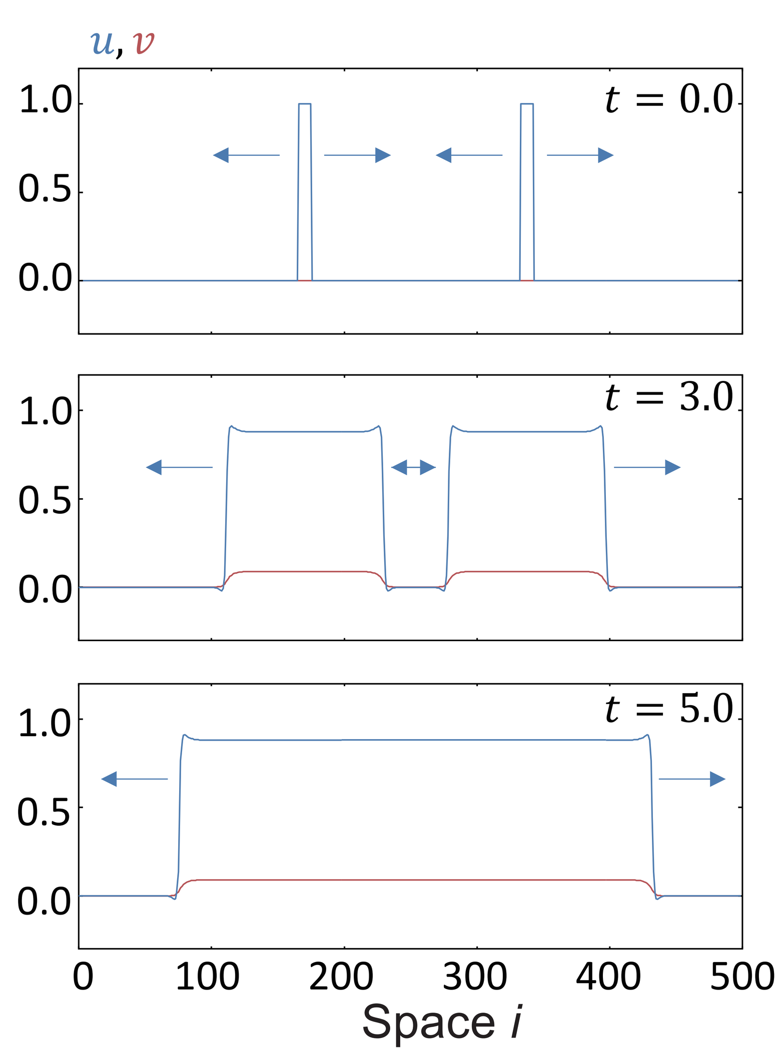
3. Phase Plot[3]

A,B: stable and C: unstable



Temporal developments

4. Coupled Nonlinear Elements



Coupled elements on a one-dimensional grid system generate propagating waves and fill-in neighboring space.

5. Proposed Algorithm

Nonlinear elements on a three-dimensional grid (i,j,k)

$$\frac{du_{i,j,k}}{dt} = C_u \left[\overline{u}\Big|_{i,j,k} - 6u_{i,j,k} \right] + f\left(u_{i,j,k}, v_{i,j,k}, a_{i,j,k}\right) + \mu S_{i,j,k}$$

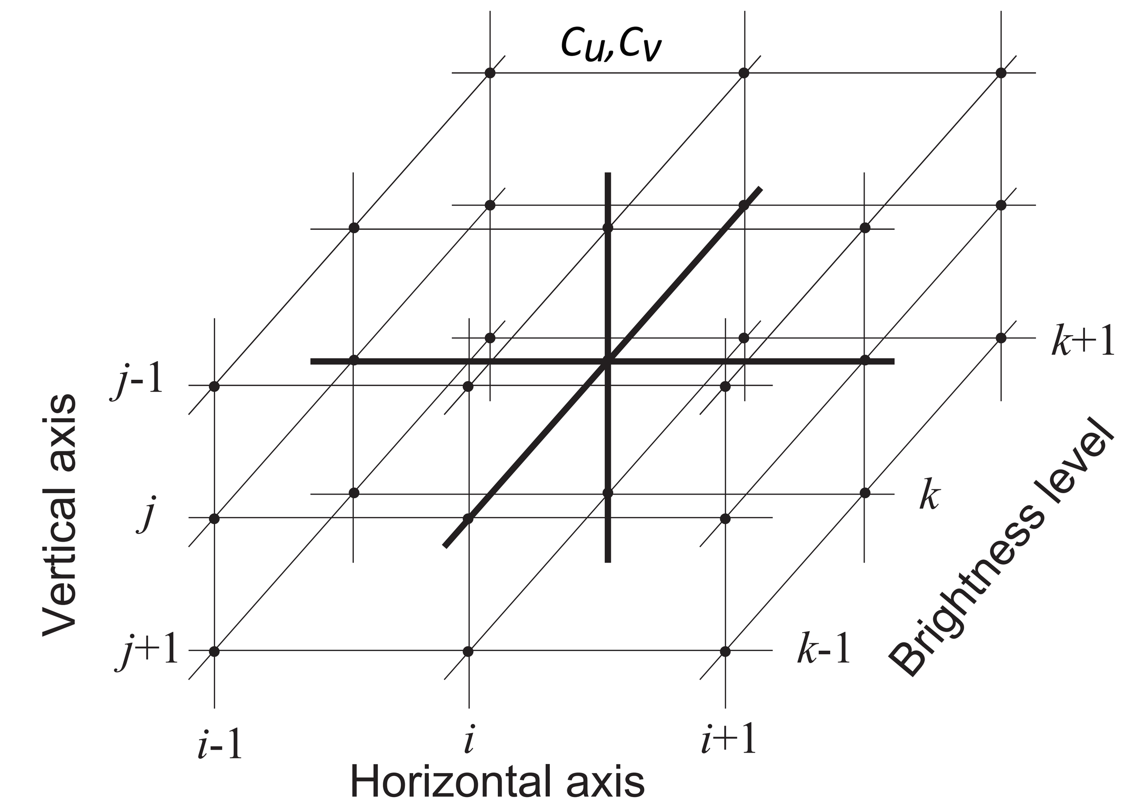
$$\frac{dv_{i,j,k}}{dt} = C_v \left[\overline{v}\Big|_{i,j,k} - 6v_{i,j,k} \right] + g\left(u_{i,j,k}, v_{i,j,k}\right)$$

$\overline{u}$ $\overline{v}$: sum of nearest neighboring elements

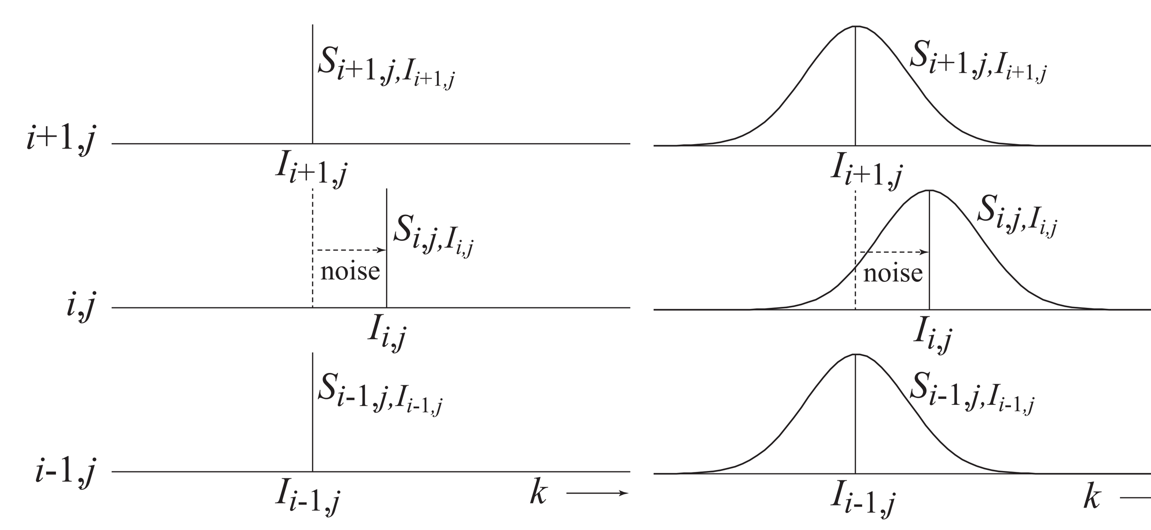
$$a_{i,j,k} = A_0 + \frac{A_2 - A_0}{2} \times \left[1 + \tanh\left(\max_{k' \in Z \setminus \{k\}} u_{i,j,k'}\right) - A_1\right]$$

Denoised image:

$$I^*_{i,j} = \mathrm{argmax}_{k \in Z} u_{i,j,k}$$



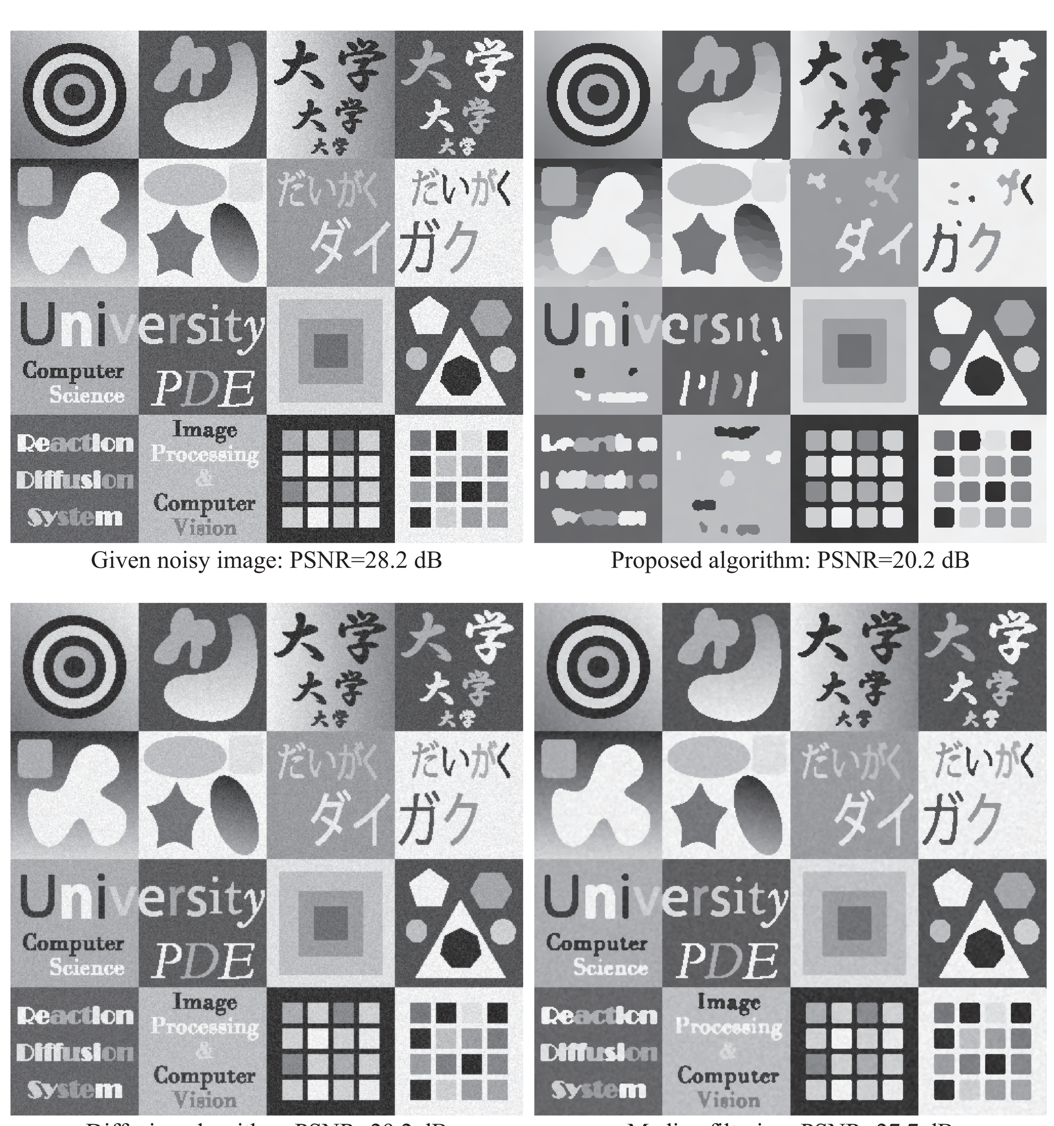
Three-dimensional grid system in the proposed algorithm



External stimulus:

$$S_{i,j,k} = \exp\left(-\frac{\left(k - I_{i,j}\right)^2}{\sigma^2}\right)$$

6. Results on an Artificial Image



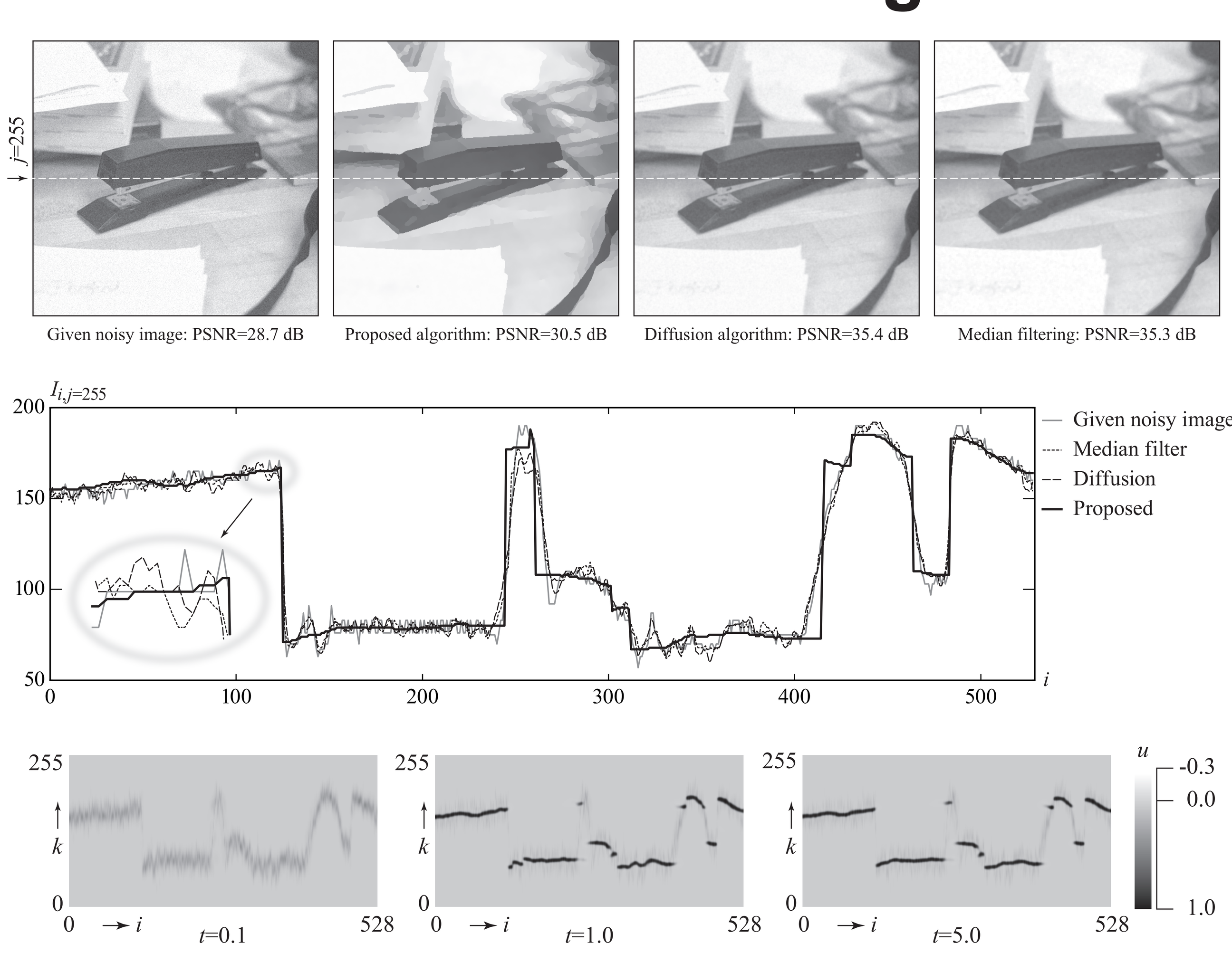
Given noisy image: PSNR=28.2 dB
Proposed algorithm: PSNR=20.2 dB
Diffusion algorithm: PSNR=29.2 dB
Median filtering: PSNR=27.7 dB

Evaluation measure:

$$\mathrm{PSNR} = 10\log_{10}\left[\frac{N^2|X \times Y|}{\sum_{i,j \in X \times Y}\left(I^t_{i,j} - I^o_{i,j}\right)^2}\right] \text{[dB]}$$

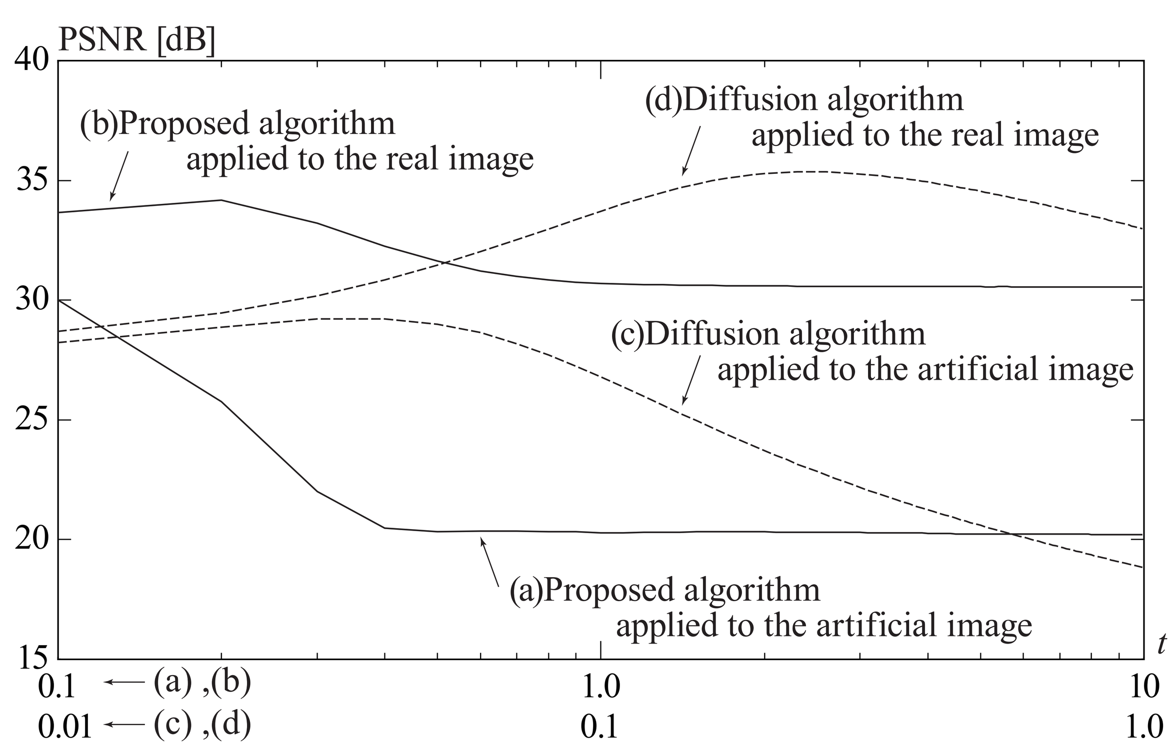
$N=2^8$; $|X \times Y|$: Image size; $I^t_{i,j}$: True image; $I^o_{i,j}$: Obtained image

7. Results on a Real Image



Given noisy image: PSNR=28.7 dB
Proposed algorithm: PSNR=30.5 dB
Diffusion algorithm: PSNR=35.4 dB
Median filtering: PSNR=35.3 dB

8. Convergence



9. Conclusion

This poster presented an image denoising algorithm. The algorithm is composed of coupled nonlinear elements in a three-dimensional grid system. A PSNR measure evaluated the performance of the proposed algorithm in comparison with previous algorithms. Although the performance is not satisfactory, the algorithm has a merit of image denoising around edges without over-smoothing, or with preserving edges. Further development must be devoted to improve the overall performance of the proposed algorithm.

References

1) R. FitzHugh, Impulses and physiological states in theoretical models of nerve membrane, Biophys. J. 1 (1961) 445-466.
2) J. Nagumo, S. Arimoto, S. Yoshizawa, An active pulse transmission line simulating nerve axon, Proc. IRE 50 (1962) 2061-2070.
3) J. D. Murray, Mathematical Biology, Springer-Verlag, Berlin, 1989.