Image Denoising Algorithm with a Three-Dimensional Grid System of Coupled Nonlinear Elements
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1. Abstract
This poster presents an image denoising algorithm with a three-dimensional grid system of time-evolving nonlinear elements. At each grid point, a nonlinear element is placed and coupled with its first nearest neighboring elements. We confirm performance and convergence of the algorithm on artificial and real images in comparison with two classical algorithms of a diffusion equation and median filtering.

2. Nonlinear Element
\[
\frac{du}{dt} = f(u, v) = \frac{1}{\epsilon} [u(u-a)(1-u)-v]
\]
\[
\frac{dv}{dt} = g(u, v) = u - bv
\]
\[(u, v): \text{variables defined at time } t > 0, f(x, y), g(x, y): \text{reaction functions, } a, b, \epsilon: \text{constants}\]
The nonlinear element behaves excitable or oscillatory, depending on its parameter settings (see the next phase plot).

3. Phase Plot

4. Coupled Nonlinear Elements

5. Proposed Algorithm
Nonlinear elements on a three-dimensional grid \((i,j,k)\)
\[
\frac{d u_{i,j,k}}{d t} = C_u \left[ u_{i,j,k} - 6u_{i,j,k} \right] + f(u_{i,j,k}, v_{i,j,k}, a_{i,j,k}) + \mu S_{i,j,k}
\]
\[
\frac{d v_{i,j,k}}{d t} = C_v \left[ v_{i,j,k} - 6v_{i,j,k} \right] + g(u_{i,j,k}, v_{i,j,k})
\]
\[a_{i,j,k} = A_0 + \frac{A_2 - A_0}{2} \times \left[ 1 + \tanh(\max_{k' \in Z(k)} u_{i,j,k'}) - A_1 \right]
\]
Denoised image:
\[I_{i,j}^* = \arg\max_{k \in Z} u_{i,j,k}\]

6. Results on an Artificial Image

7. Results on a Real Image

8. Convergence

9. Conclusion
This poster presents an image denoising algorithm. The algorithm is composed of coupled nonlinear elements in a three-dimensional grid system. A PSNR measure evaluated the performance of the proposed algorithm in comparison with previous algorithms. Although the performance is not satisfactory, the algorithm has a merit of image denoising around edges without over-smoothing, or with preserving edges. Further development must be devoted to improve the overall performance of the proposed algorithm.

References

Evaluation measure:
\[\text{PSNR} = 10 \log_{10} \left( \frac{N^2}{\sum_{(i,j) \in Z} (I_{i,j} - I_{i,j}^*)^2} \right) \text{ dB}\]

Temporal developments

P0: $I_{i,j}^*$
A, B: stable and C: unstable

C: $S_{i,j,k}$

A: $C_u$, B: $C_v$, C: $\mu$

Nonlinear elements on a three-dimensional grid system generate propagating waves and fill-in neighboring space.

Denoised image:
\[I_{i,j}^* = \arg\max_{k \in Z} u_{i,j,k}\]