Image Edge Detection Algorithm with a Single Grid System of Coupled FitzHugh-Nagumo Elements

A. Nomura, M. Ichikawa, K. Okada,
T. Sakurai & Y. Mizukami
Yamaguchi Univ. & Chiba Univ.
Japan

Key words: image processing, edge detection
application of nonlinear elements, FitzHugh-Nagumo model
initial conditions for FHN model

Outline

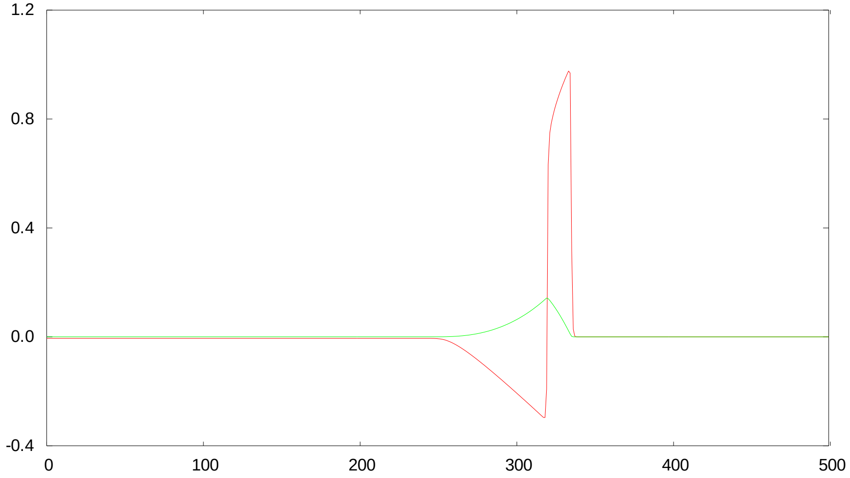
- Introduction: Motivation and Approach
- Background:
  - Nonlinear phenomena and pattern formation in nature
  - Image edge detection and previous algorithms in image processing
- Our Previous Edge Detection Algorithm with cFHN
  - FitzHugh-Nagumo (FHN) model
  - Grid system of coupled FHN (cFHN) and the initial conditions
- Proposed Edge Detection Algorithm with cFHN
  - The initial conditions
- Experimental Results
  - Artificial images with/without noise
  - Real images
- Conclusion

Introduction: Motivation and Approach

- Motivation
  - Biological visual system
  - Bio-inspired image processing
    - edge detection, segmentation and stereo disparity detection
- Approach
  - Coupled FitzHugh-Nagumo (cFHN) elements on a grid system
  - Reaction-diffusion system (diffusively coupled elements)
- Our previous edge detection algorithm
  - does not work for gray level image
  - noise vulnerability or not robust to noise

Background:
Nonlinear Phenomena & Pattern Formation

- Nonlinear elements in nature
  - Biological response to external stimuli: FitzHuhg-Nagumo model
  - Nonlinear oscillation or excitation in chemical reaction system
- Reaction-diffusion system
  - System of diffusively coupled nonlinear elements in space
  - Patterns: traveling pulses in 1D space and spiral waves in 2D space
  - Information transmission & information processing

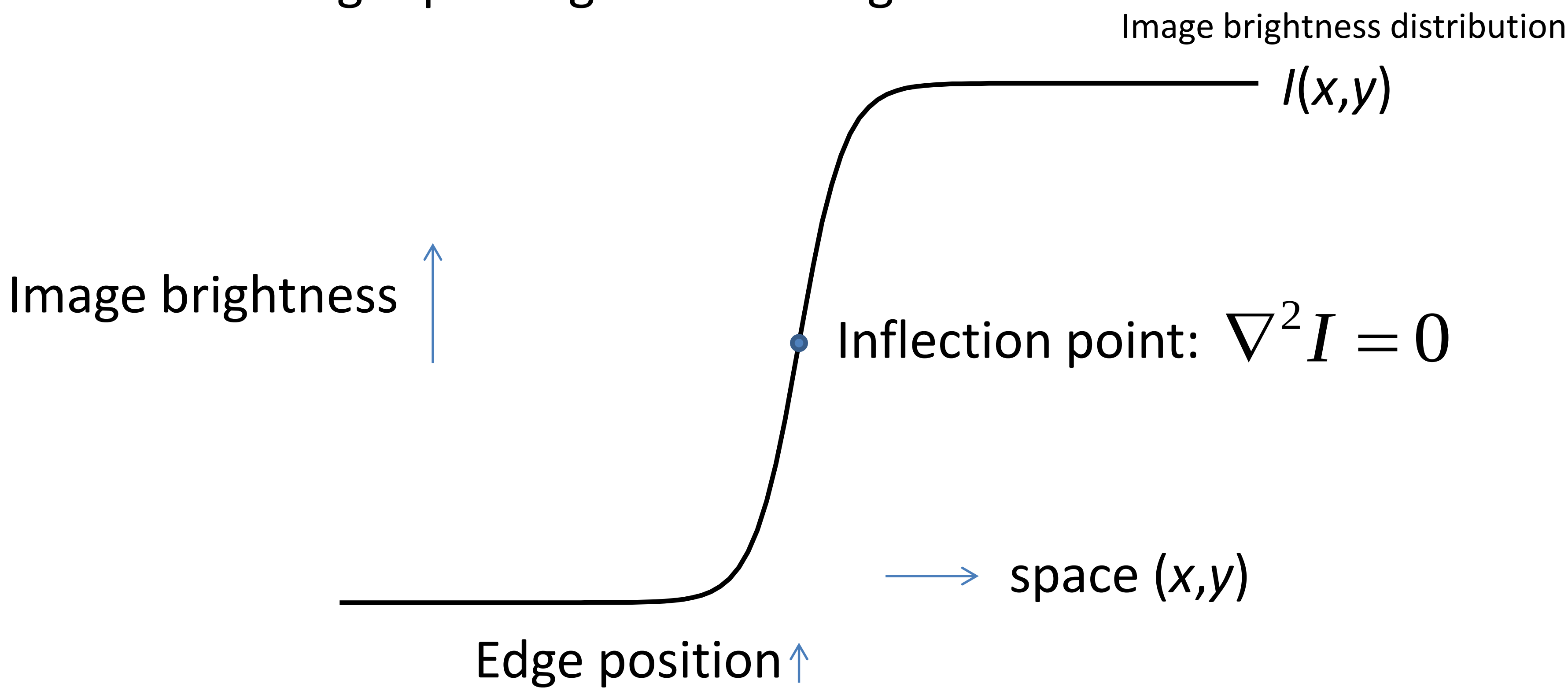


1D pulse propagating in space

2D spiral waves

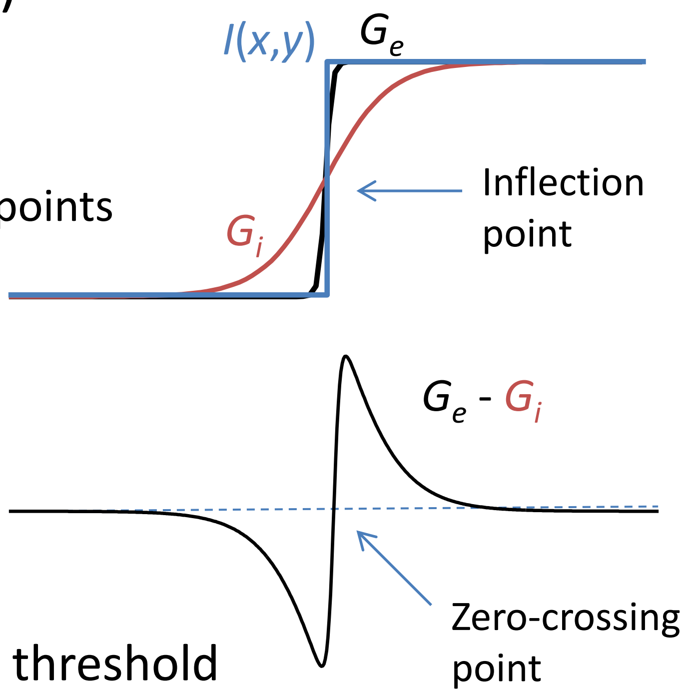
Background: Definition of Image Edge

- Point having rapid brightness change



Background: Previous Edge Detection Algorithms

- Algorithms by Marr and Hildreth (1980)
  - LoG: Laplacian-of-the-Gaussian
    - Gaussian: noise reduction
    - Laplacian operator: detection of inflection points
  - DoG: Difference-of-two-Gaussians
    - G_e: excitation (blurred)
    - G_i: inhibition (more blurred)
  - Detecting zero-crossing points
- Algorithm by Canny (1986)
  - Gaussian smoothing + gradient operator + threshold
  - assumption: continuity of edges



Our Previous Edge Detection Algorithm: FitzHugh-Nagumo (FHN) Model

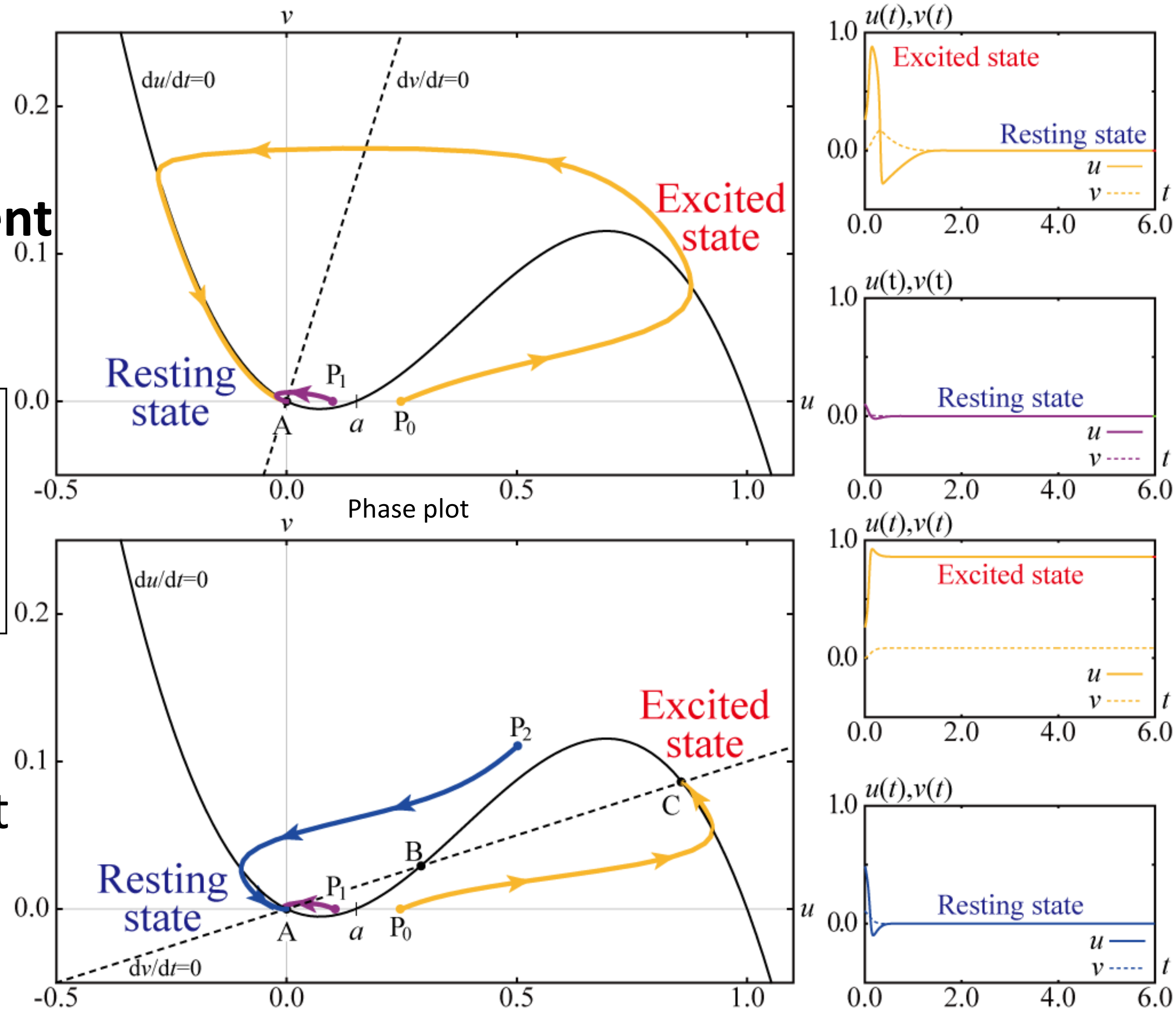
(a) **Uni-stable element**

FHN model

$$\frac{du}{dt} = \frac{1}{\varepsilon}\left[u(u-a)(1-u)-v\right]$$

$$\frac{dv}{dt} = u - bv$$

(b) Bi-stable element



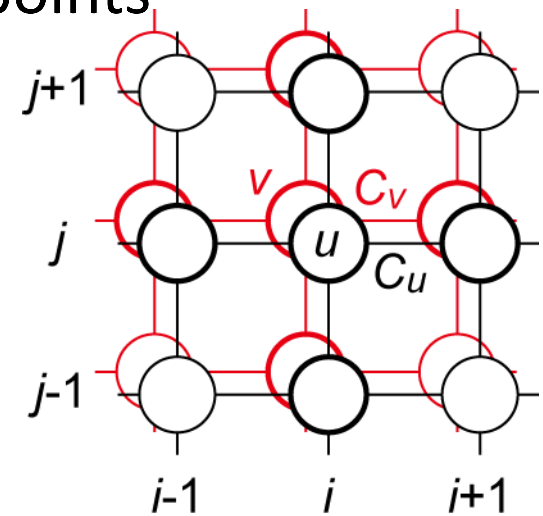
Our Previous Edge Detection Algorithm: Single Grid System of Coupled FHN (cFHN)

- Uni-stable elements placed at image grid points
  - Nomura et al., *J. Phys. Soc. Jpn.*, 2003
  - Kurata et al., *Phys. Rev. E*, 2009

$$\frac{\mathrm{d}u_{i,j}}{\mathrm{d}t} = C_u\left[\overline{u_{i,j}} - 4u_{i,j}\right] + \frac{1}{\varepsilon}\left[u_{i,j}(u_{i,j}-a)(1-u_{i,j}) - v_{i,j}\right]$$

$$\frac{\mathrm{d}v_{i,j}}{\mathrm{d}t} = C_v\underbrace{\left[\overline{v_{i,j}} - 4v_{i,j}\right]}_{\text{Spatial coupling}} + u_{i,j} - bv_{i,j}$$

$\overline{u_{i,j}}, \overline{u_{i,j}}$: averages in the nearest four points.



- The initial conditions:

  $u_{i,j}=I_{i,j}$, $v_{i,j}=0$

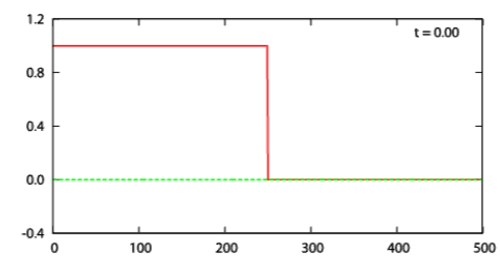
- Strong inhibition: $C_u \ll C_v$

  ⇒ Stationary pulses at edge positions

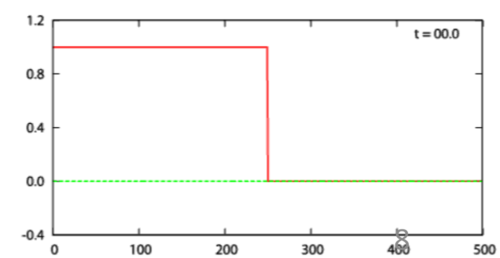
- Weak inhibition: $C_u > C_v$

  ⇒ Propagating pulses

u and *v* ($C_u \ll C_v$)



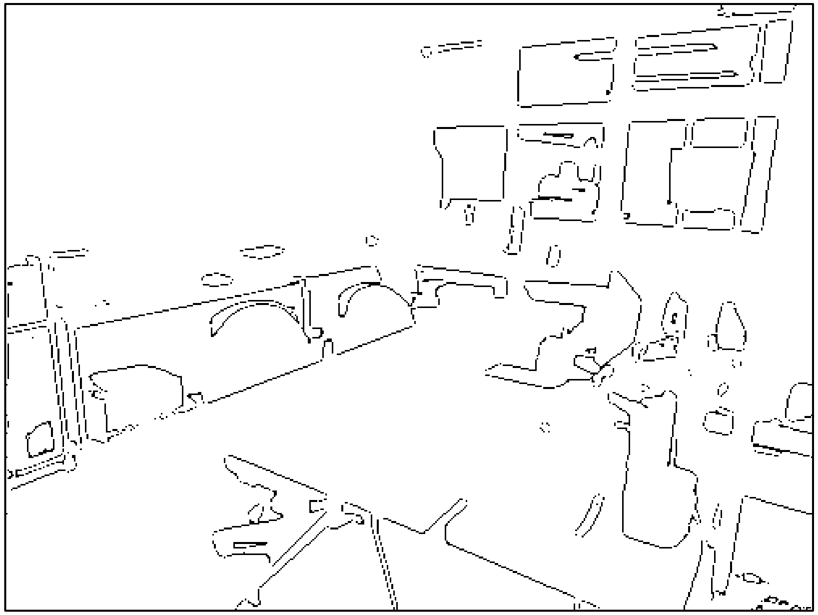
u and *v* ($C_u > C_v$)



Our Previous Algorithm for Edge Detection: Example of Edge Detection with cFHN

- Example:

Initial condition of $u_{i,j}$



Result of edge detection (a=0.1)

- Threshold for the initial condition u_0 & Self-organized pulse
  => Previous algorithm does not work for gray level images

Proposed Algorithm: cFHN & Initial Conditions

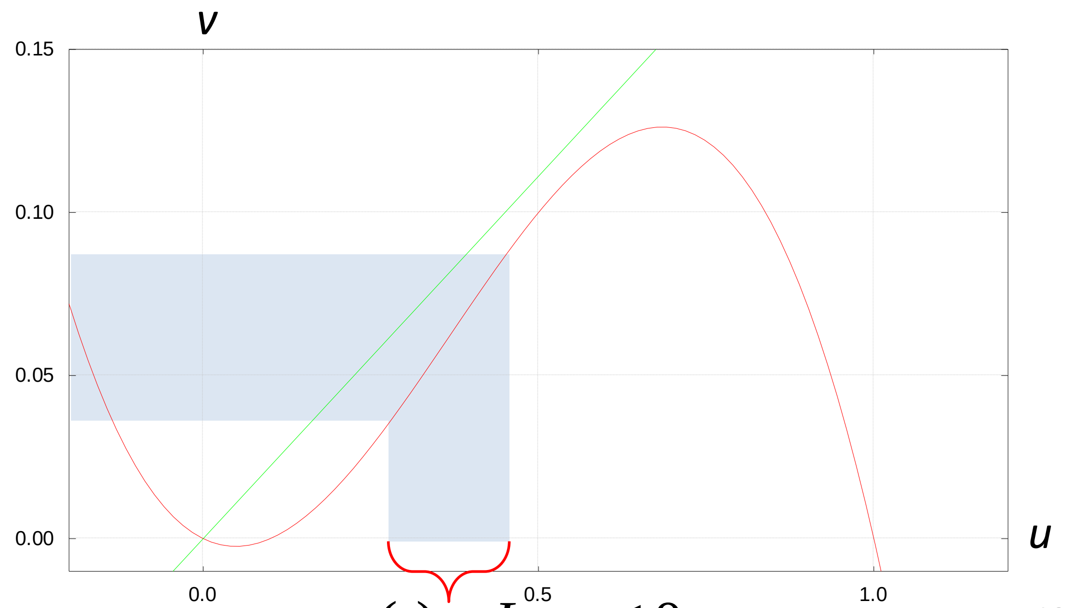
- Coupled FHN elements: delaying computation of $u_{i,j}$

$$\frac{\mathrm{d}u_{i,j}}{\mathrm{d}t} = C_u\left[\overline{u_{i,j}} - 4u_{i,j}\right] + \underbrace{\frac{1}{\varepsilon}\left[u_{i,j}(u_{i,j} - a)(1 - u_{i,j}) - v_{i,j}\right]}_{f(u_{i,j},\, v_{i,j})}, \quad \underline{t > 0}$$

$$\frac{\mathrm{d}v_{i,j}}{\mathrm{d}t} = C_v\left[\overline{v_{i,j}} - 4v_{i,j}\right] + u_{i,j} - bv_{i,j}, \quad \underline{t > -\tau}$$

- The initial conditions

$$v_{i,j}(t = -\tau) = f(I_{i,j}, 0)$$



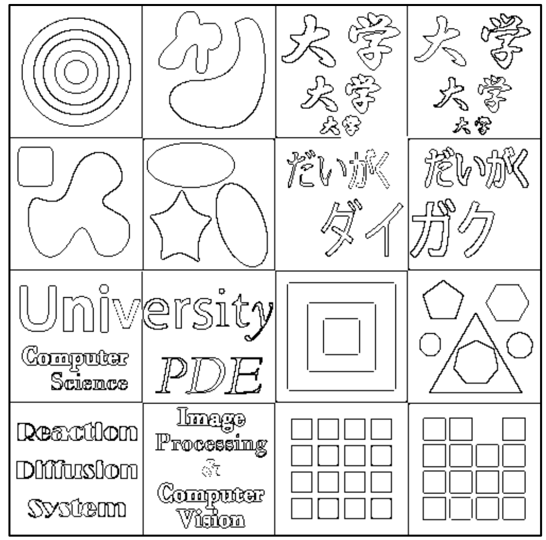
$$u_{i,j}(t) = I_{i,j},\, t \le 0$$

Experimental Results: Artificial Noiseless Image

The Image

500×500 pixels

256 brightness levels

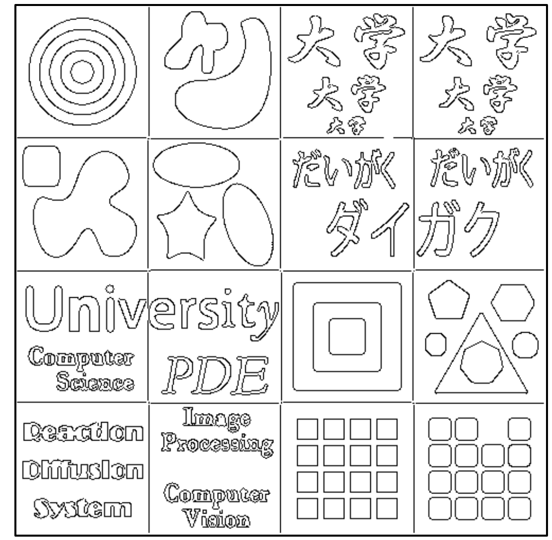


Proposed Algorithm

C_u=4,C_v=12, a=0.1, b=4.5,

ε=1.0×10^{-3}, τ=5.0 ×10^{-4}

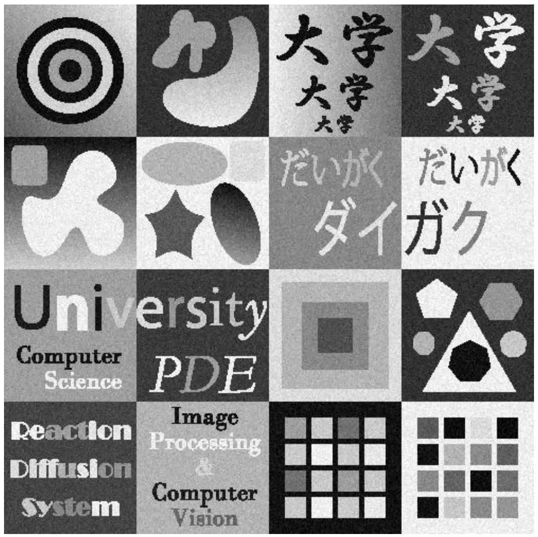
δt=1.0 × 10^{-4}

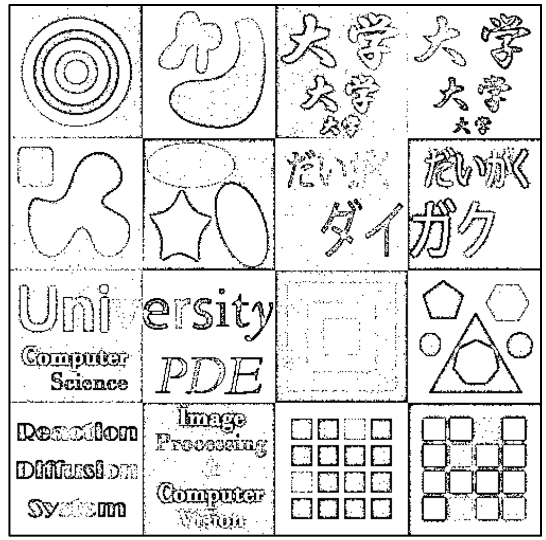


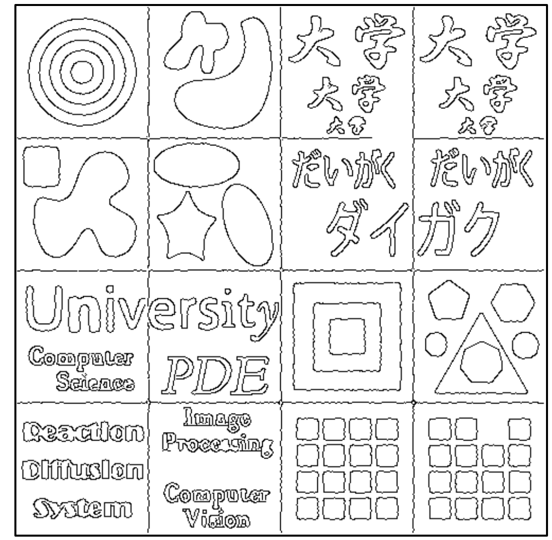
Canny Algorithm

σ=0.40, θ_l=0.10, θ_h=0.20

Experimental Results: Artificial Noisy Image







The Image
500×500 pixels
256 brightness levels
S.D. of noise=10

Proposed Algorithm
C_u=4,C_v=12, a=0.1, b=4.5,
ε=1.0×10^{-3}, τ=0.1
δt=1.0 × 10^{-4}

Canny Algorithm
σ=1.20
θ_l=0.40, θ_h=0.70

Experimental Results:
Quantitative Results with P, R and F measures

Algorithm	Image	P	R	F
Our Previous Algorithm	(a) (b)	0.989 0.747	0.906 0.908	**0.946** **0.819**
Proposed Algorithm	(a) (b)	0.999 0.825	0.979 0.945	**0.989** **0.881**
Canny Algorithm	(a) (b)	1.000 0.999	0.975 0.965	**0.987** **0.982**

Image:
(a) Noiseless image
(b) Noisy image (s.d.=10.0)

red is the best performance

- Algorithms:
  - Our Previous Algorithm (Nomura et al., 2011)
  - Proposed Algorithm
  - Canny Algorithm (Canny, 1986)
- Evaluation measures:

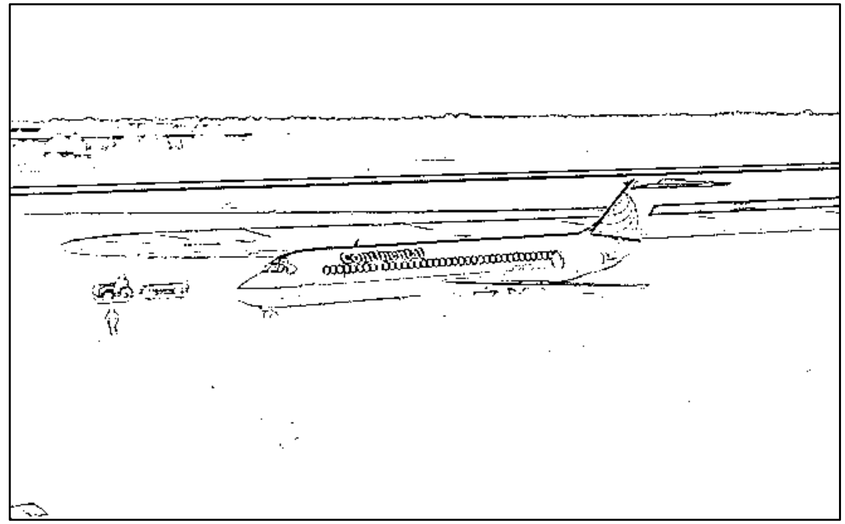
  M_o: obtained edge map

  M_t: true edge map

$$P = \frac{|M_t \cap M_o|}{|M_o|}, R = \frac{|M_t \cap M_o|}{|M_t|}, F = \frac{2PR}{P+R}$$

Experimental Results: Real Image (1/2)

The image
659×409 pixels, 256 brightness levels
http://marathon.csee.usf.edu/edge/edgecompare_main.html

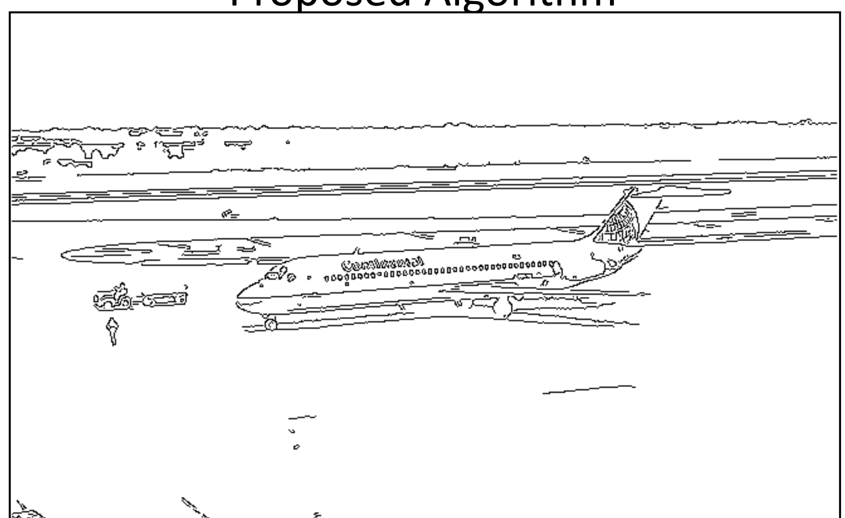


Proposed Algorithm

C_u=4
C_v=12
a=0.1
b=4.5
ε=1.0×10^{-3}
τ=0.1
δt=1.0 × 10^{-4}

Proposed Algorithm

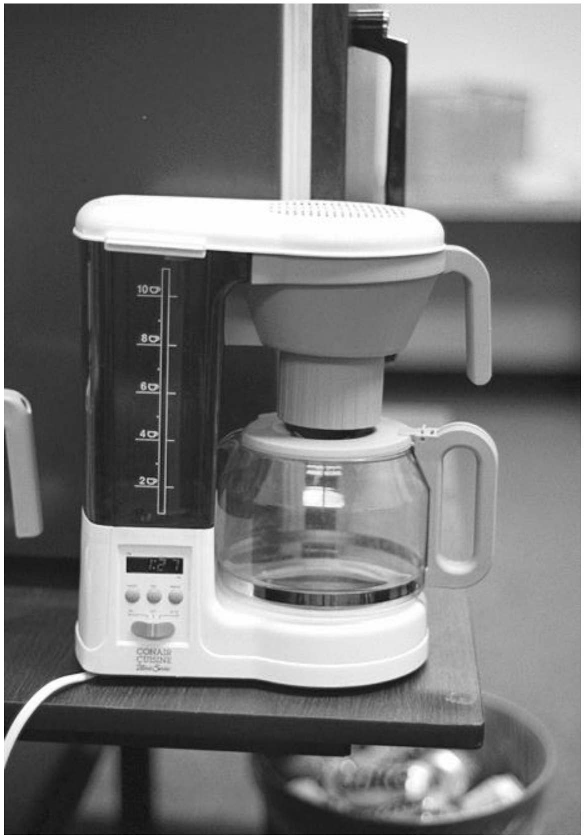
Canny Algorithm



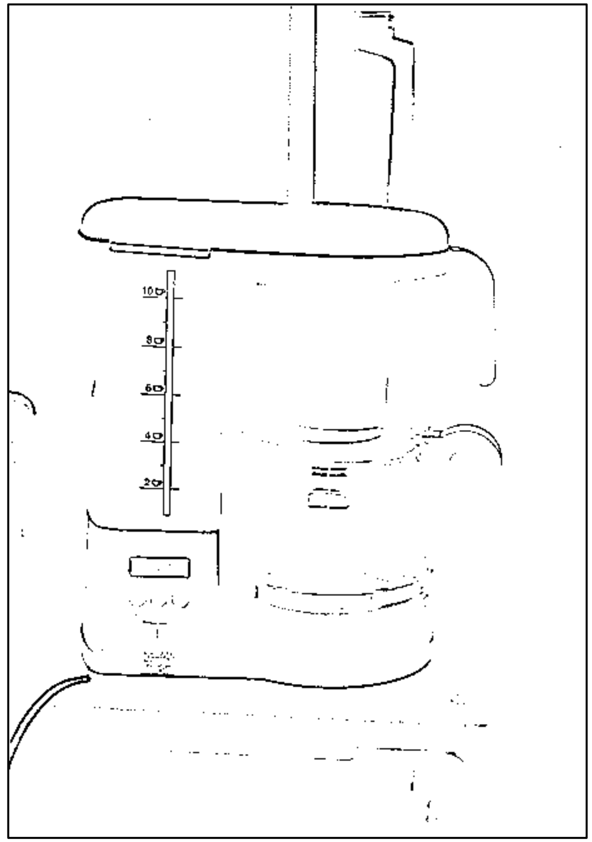
Canny Algorithm
http://marathon.csee.usf.edu/edge/edgecompare_main.html

σ=0.6
θ_l=0.5
θ_h=0.9

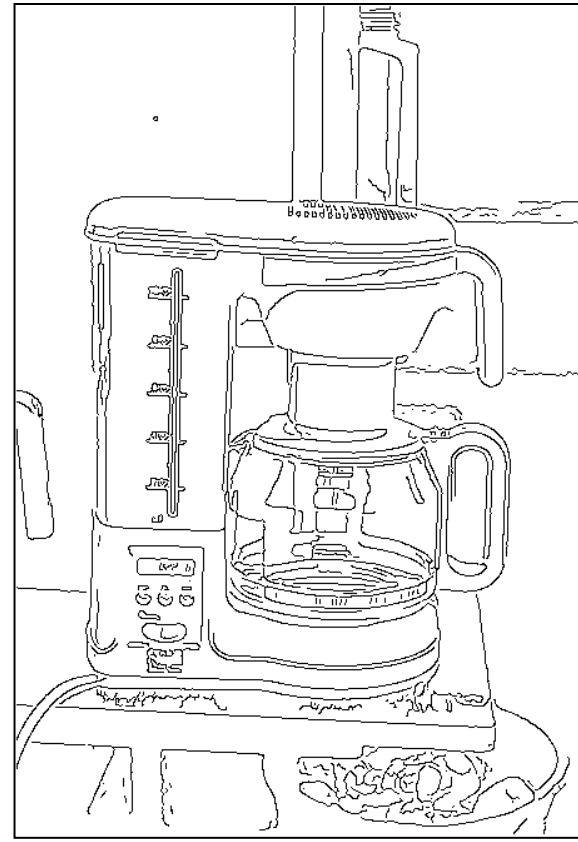
Experimental Results: Real Image (2/2)



The image
461×665 pixels
256 brightness levels
http://marathon.csee.usf.edu/edge/edgecompare_main.html



Proposed Algorithm
$C_u=4, C_v=12$, $a=0.1$, $b=4.5$
$\varepsilon=1.0\times10^{-3}$, $\tau=0.1$
$\delta t=1.0 \times 10^{-4}$



Canny Algorithm
$\sigma=1.2$, $\theta_l=0.3$, $\theta_h=0.8$
http://marathon.csee.usf.edu/edge/edgecompare_main.html

Conclusion

- Grid system of coupled FitzHugh-Nagumo elments for image edge detection
  - Reconsidering initial conditions for $u_{i,j}$ and $v_{i,j}$
  - Delaying computation of $u_{i,j}$
- Experiments for artificial and real gray level images
- The proposed algorithm achieved better performance than our previous algorithm.
- Future topics:
  - Noise robustness
  - Detection of blurred edges and edge strength evaluation