Image Edge Detection Algorithm with a Single Grid System of Coupled FitzHugh-Nagumo Elements

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Key words: image processing, edge detection, application of nonlinear elements, FitzHugh-Nagumo model, initial conditions for FHN model
Outline

• Introduction: Motivation and Approach
• Background:
  – Nonlinear phenomena and pattern formation in nature
  – Image edge detection and previous algorithms in image processing
• Our Previous Edge Detection Algorithm with cFHN
  – FitzHugh-Nagumo (FHN) model
  – Grid system of coupled FHN (cFHN) and the initial conditions
• Proposed Edge Detection Algorithm with cFHN
  – The initial conditions
• Experimental Results
  – Artificial images with/without noise
  – Real images
• Conclusion
Introduction: Motivation and Approach

• Motivation
  – Biological visual system
  – Bio-inspired image processing
    • edge detection, segmentation and stereo disparity detection

• Approach
  – Coupled FitzHugh-Nagumo (cFHN) elements on a grid system
  – Reaction-diffusion system (diffusively coupled elements)

• Our previous edge detection algorithm
  – does not work for gray level image
  – noise vulnerability or not robust to noise
Background: Nonlinear Phenomena & Pattern Formation

• Nonlinear elements in nature
  – Biological response to external stimuli: FitzHugh-Nagumo model
  – Nonlinear oscillation or excitation in chemical reaction system

• Reaction-diffusion system
  – System of diffusively coupled nonlinear elements in space
  – Patterns: traveling pulses in 1D space and spiral waves in 2D space
  – Information transmission & information processing
Background: Definition of Image Edge

- Point having rapid brightness change

\[ \nabla^2 I = 0 \]

Image brightness distribution

Image brightness

Inflection point: \( \nabla^2 I = 0 \)

Edge position

Space \((x,y)\)
Background: Previous Edge Detection Algorithms

- Algorithms by Marr and Hildreth (1980)
  - LoG: Laplacian-of-the-Gaussian
    - Gaussian: noise reduction
    - Laplacian operator: detection of inflection points
  - DoG: Difference-of-two-Gaussians
    - $G_e$: excitation (blurred)
    - $G_i$: inhibition (more blurred)
  - Detecting zero-crossing points

- Algorithm by Canny (1986)
  - Gaussian smoothing + gradient operator + threshold
  - assumption: continuity of edges
Our Previous Edge Detection Algorithm: FitzHugh-Nagumo (FHN) Model

(a) Uni-stable element

FHN model
\[
\frac{du}{dt} = \frac{1}{\varepsilon} \left[ u(u-a)(1-u) - v \right]
\]
\[
\frac{dv}{dt} = u - bv
\]

(b) Bi-stable element

Phase plot
Our Previous Edge Detection Algorithm:
Single Grid System of Coupled FHN (cFHN)

• Uni-stable elements placed at image grid points

\[
\begin{align*}
\frac{du_{i,j}}{dt} &= C_u \left[u_{i,j} - 4u_{i,j}\right] + \frac{1}{\varepsilon} \left[u_{i,j} (u_{i,j} - a)(1-u_{i,j}) - v_{i,j}\right] \\
\frac{dv_{i,j}}{dt} &= C_v \left[v_{i,j} - 4v_{i,j}\right] + u_{i,j} - bv_{i,j} \\
\end{align*}
\]

Spatial coupling \( \bar{u}_{i,j}, \bar{u}_{i,j} \): averages in the nearest four points.

• The initial conditions:
  \( u_{i,j} = l_{i,j}, \ v_{i,j} = 0 \)

• Strong inhibition: \( C_u << C_v \)
  \( \Rightarrow \) Stationary pulses at edge positions

• Weak inhibition: \( C_u > C_v \)
  \( \Rightarrow \) Propagating pulses
Our Previous Algorithm for Edge Detection: Example of Edge Detection with cFHN

- Example:

  - Threshold for the initial condition $u_0$ & Self-organized pulse

  => Previous algorithm does not work for gray level images
Proposed Algorithm: cFHN & Initial Conditions

- Coupled FHN elements: delaying computation of $u_{i,j}$

\[
\frac{du_{i,j}}{dt} = C_u \left[ u_{i,j} - 4u_{i,j} \right] + \frac{1}{\varepsilon} \left[ u_{i,j} (u_{i,j} - a)(1 - u_{i,j}) - v_{i,j} \right], \quad t > 0
\]

\[
f(u_{i,j}, v_{i,j})
\]

\[
\frac{dv_{i,j}}{dt} = C_v \left[ v_{i,j} - 4v_{i,j} \right] + u_{i,j} - bv_{i,j}, \quad t > -\tau
\]

- The initial conditions

\[
v_{i,j}(t = -\tau) = f(I_{i,j}, 0)
\]
Experimental Results: Artificial Noiseless Image

The Image  
500×500 pixels  
256 brightness levels

Proposed Algorithm  
\( C_u = 4, C_v = 12, \sigma = 0.1, b = 4.5, \varepsilon = 1.0 \times 10^{-3}, \tau = 5.0 \times 10^{-4}, \delta t = 1.0 \times 10^{-4} \)

Canny Algorithm  
\( \sigma = 0.40, \theta_l = 0.10, \theta_h = 0.20 \)
Experimental Results: Artificial Noisy Image

The Image
500×500 pixels
256 brightness levels
S.D. of noise=10

Proposed Algorithm
\( C_u=4, C_v=12, \alpha=0.1, b=4.5, \varepsilon=1.0 \times 10^{-3}, \tau=0.1 \)
\( \delta t=1.0 \times 10^{-4} \)

Canny Algorithm
\( \sigma=1.20 \)
\( \theta_l=0.40, \theta_h=0.70 \)
Experimental Results:
Quantitative Results with P, R and F measures

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Image</th>
<th>P</th>
<th>R</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Our Previous Algorithm</td>
<td>(a)</td>
<td>0.989</td>
<td>0.906</td>
<td>0.946</td>
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<tr>
<td></td>
<td>(b)</td>
<td>0.747</td>
<td>0.908</td>
<td>0.819</td>
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<tr>
<td>Proposed Algorithm</td>
<td>(a)</td>
<td>0.999</td>
<td>0.979</td>
<td>0.989</td>
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<tr>
<td></td>
<td>(b)</td>
<td>0.825</td>
<td>0.945</td>
<td>0.881</td>
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<tr>
<td>Canny Algorithm</td>
<td>(a)</td>
<td>1.000</td>
<td>0.975</td>
<td>0.987</td>
</tr>
<tr>
<td></td>
<td>(b)</td>
<td>0.999</td>
<td>0.965</td>
<td>0.982</td>
</tr>
</tbody>
</table>

Image: (a) Noiseless image, (b) Noisy image (s.d.=10.0)

red is the best performance

• Algorithms:
  – Our Previous Algorithm (Nomura et al., 2011)
  – Proposed Algorithm
  – Canny Algorithm (Canny, 1986)

• Evaluation measures:
  \[ P = \frac{|M_t \cap M_o|}{|M_o|}, \quad R = \frac{|M_t \cap M_o|}{|M_t|}, \quad F = \frac{2PR}{P + R} \]

\( M_o \): obtained edge map
\( M_t \): true edge map
Experimental Results: Real Image (1/2)

The image
659×409 pixels, 256 brightness levels
http://marathon.csee.usf.edu/edge/edgecompare_main.html

Proposed Algorithm

- $C_u = 4$
- $C_v = 12$
- $a = 0.1$
- $b = 4.5$
- $\varepsilon = 1.0 \times 10^{-3}$
- $\tau = 0.1$
- $\delta t = 1.0 \times 10^{-4}$

Canny Algorithm

- $\sigma = 0.6$
- $\theta_l = 0.5$
- $\theta_h = 0.9$

http://marathon.csee.usf.edu/edge/edgecompare_main.html
Experimental Results: Real Image (2/2)

The image
461×665 pixels
256 brightness levels
http://marathon.csee.usf.edu/edge/edgecompare_main.html

Proposed Algorithm

$C_u = 4, C_v = 12, \alpha = 0.1, b = 4.5$
$\varepsilon = 1.0 \times 10^{-3}, \tau = 0.1$
$\delta t = 1.0 \times 10^{-4}$

Canny Algorithm

$\sigma = 1.2, \theta_l = 0.3, \theta_h = 0.8$
http://marathon.csee.usf.edu/edge/edgecompare_main.html
Conclusion

• Grid system of coupled FitzHugh-Nagumo elements for image edge detection
  – Reconsidering initial conditions for $u_{i,j}$ and $v_{i,j}$
  – Delaying computation of $u_{i,j}$
• Experiments for artificial and real gray level images
• The proposed algorithm achieved better performance than our previous algorithm.
• Future topics:
  – Noise robustness
  – Detection of blurred edges and edge strength evaluation