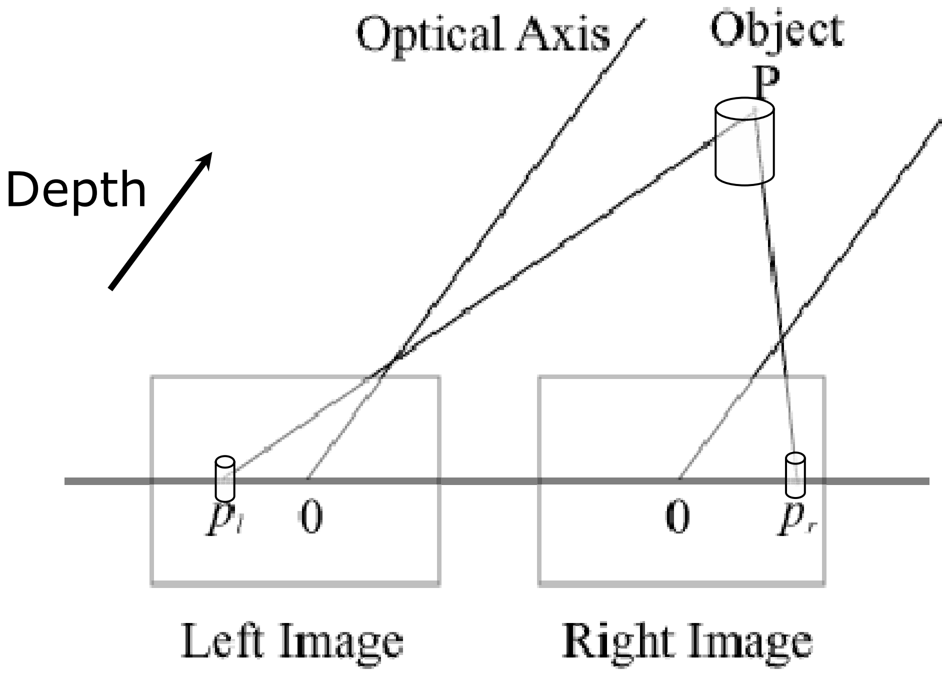
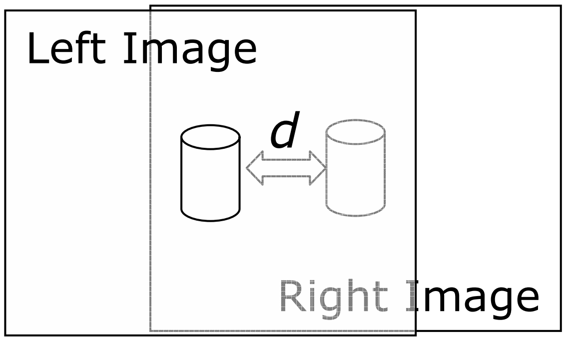
SUMMARY

We propose a model detecting stereo disparity. The model utilizes multi-sets of reaction-diffusion equations having activator and inhibitor variables. Diffusion terms of the model realize the continuity constraint and the mutual-inhibition mechanism built in reaction terms does the uniqueness one. In addition, the ratio of two diffusion coefficients realizes the self-inhibition mechanism, which brings better results in detecting stereo disparity. Performance of the proposed model is confirmed through the analysis of several test images.

Stereo Vision System



Disparity: $d = p_l - p_r$ (pixel)



Compute a correlation map $C(x,y,d)$ from overlapped stereo images.

Find correspondence relation between stereo images.

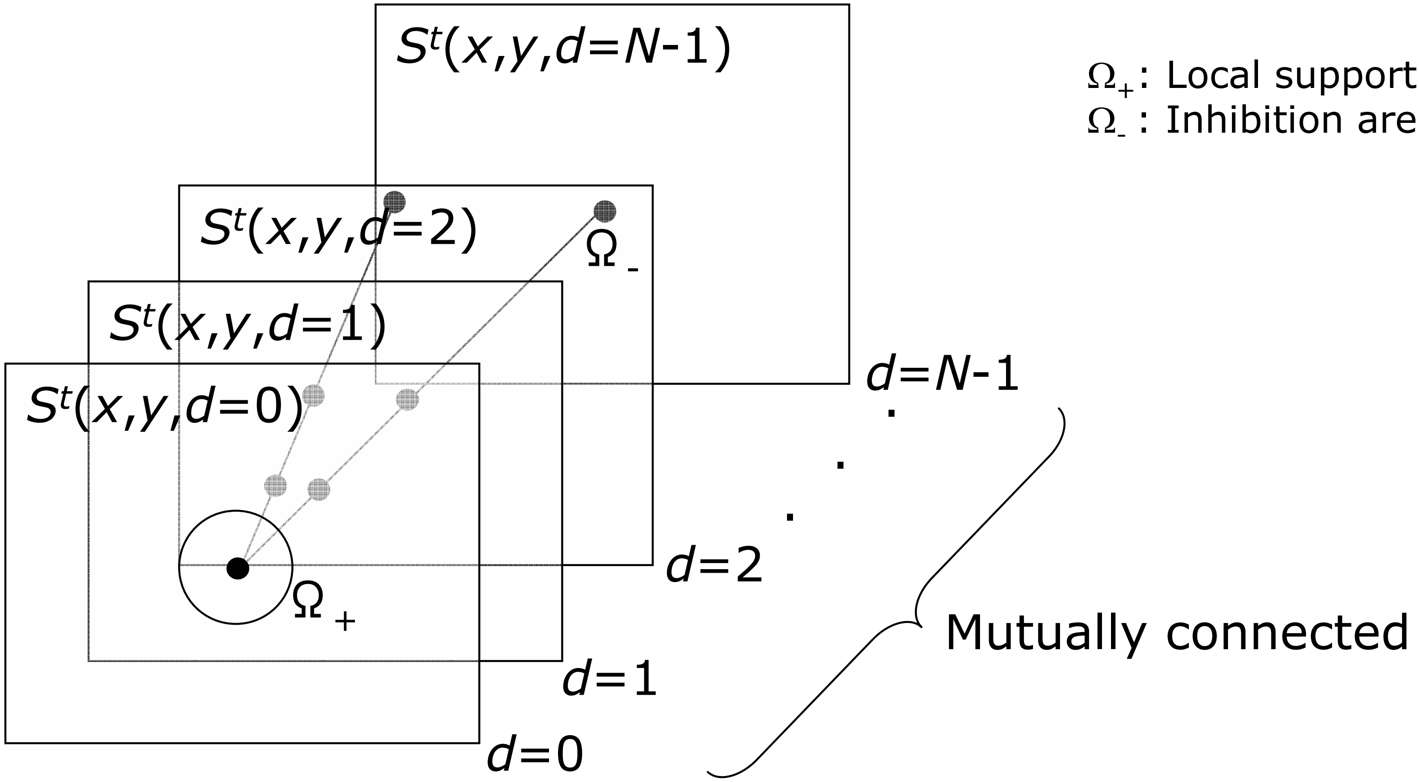
Stereo Constraints
Proposed by Marr and Poggio

- Uniqueness constraint
  - A particular point has only one disparity level.
- Continuity constraint
  - Neighboring points have similar disparity levels.

Marr, D., Poggio, T.: Cooperative computation of stereo disparity. Science Vol.194, pp.283-287 (1976)

Cooperative Model

- Multi-layered network model



Ω_+: Local support area
Ω_- : Inhibition area

Cooperative Model Proposed by Marr and Poggio

- Update function:

$$S^{t+1}(x,y,d) = \sigma\left(\sum_{\Omega_+} S^t - \omega \sum_{\Omega_-} S^t + S^0, T\right)$$

- Disparity map: $M(x,y) = \underset{d=0,1,\ldots,N-1}{\arg\max}\, S^t(x,y,d)$

t: Iterative number, $\sigma(\cdot)$: Threshold function, T: Threshold value
ω :Inhibitory constant, S^0: Correlation map $C(x,y,d)$
Ω_+: Local support area for the continuity constraint
Ω_- : Inhibitory area for the uniqueness constraint

Marr, D., Poggio, T.: Cooperative computation of stereo disparity. Science Vol.194, pp.283-287 (1976)

Cooperative Model Proposed by Zitnick and Kanade

- Update function:

$$S^{t+1}(x,y,d) = S^0 \times \left(\frac{R^t(x,y,d)}{\sum_{\Omega_-} R^t} \right)^{\alpha}$$

α : constant

$R^t = \sum_{\Omega_+} S^t$

- Disparity map:

$$M(x,y) = \begin{cases} \arg\max_{d=0,1,\ldots,N-1} S^t(x,y,d) & \text{if} \quad \max_{d=0,1,\ldots,N-1} S^t > T \\ \text{Occlusion Area} & \text{else} \end{cases}$$

Zitnick, C. L., Kanade, T.: A cooperative algorithm for stereo matching and occlusion detection. IEEE PAMI Vol.22, pp.675-684 (2000)

Original FitzHugh-Nagumo Type Reaction-Diffusion Equations

- The model:

$$\frac{\partial u}{\partial t} = D_u \nabla^2 u + \frac{1}{\varepsilon}\left[u(1-u)(u-a)-v\right] \qquad u(x,t): \text{Activator}$$

$$\frac{\partial v}{\partial t} = (u-bv) \qquad v(x,t): \text{Inhibitor}$$

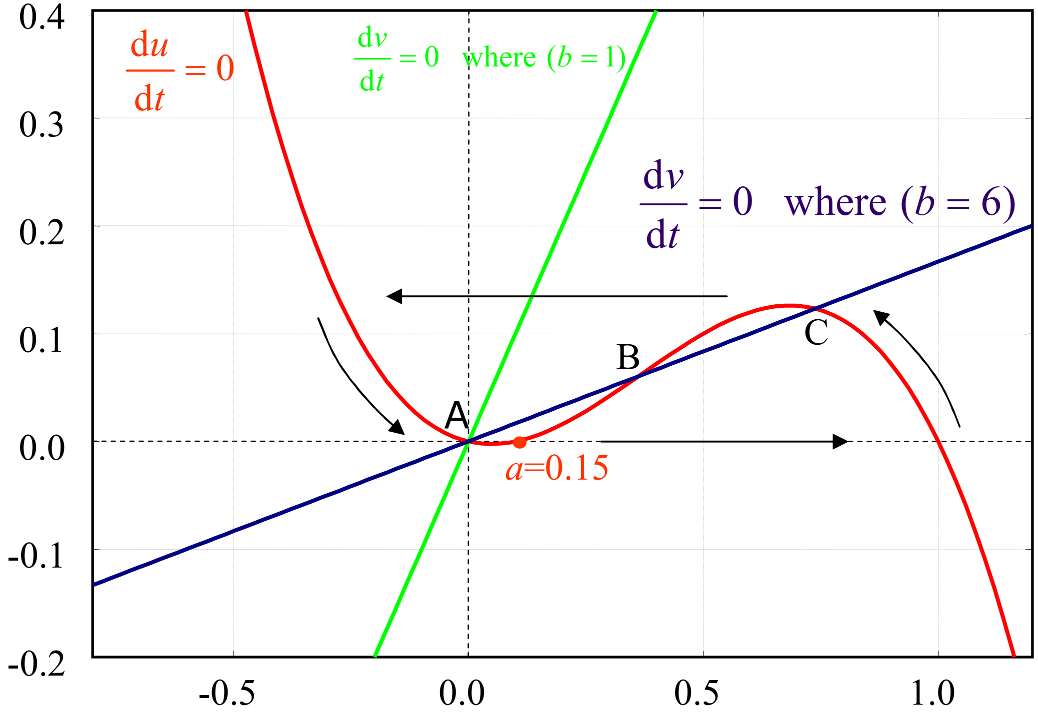
Diffusion term

Reaction terms

ε, a, b: constants

FitzHugh, R.: Impulses and physiological states in theoretical models of nerve membrane. Biophysical J. Vol.1, pp.445-466 (1961)
Nagumo, J., Arimoto, S., Yoshizawa,S.: An active pulse transmission line simulating nerve axon. Proc. IRE Vol.50, pp.2061-2070 (1962)

Null-Clines of the FitzHugh-Nagumo Equations in O.D.E.



Null-clines

$$\frac{du}{dt} = \frac{1}{\varepsilon}\{u(1-u)(u-a)-v\}$$

$$\frac{dv}{dt} = u - bv$$

Stable point:A
Uni-stable system

Stable points:A & C
Bi-stable system:

Bi-stable system => Threshold function

Revised Version of the Cooperative Model in P.D.E.

- Cooperative model utilizing reaction-diffusion equations

$$\frac{\partial S_d}{\partial t} = D\nabla^2 S_d + \frac{1}{\varepsilon} S_d (1 - S_d)(S_d - a(S_{\max})) + rC$$

D: Diffusion coefficient
ε,*r*: Constant

or

$$\frac{\partial S_d}{\partial t} = D\nabla^2 S_d + \frac{1}{\varepsilon} S_d (1 - S_d)(S_d - a_0) + r(C - S_{\max})$$

$$d = 0,1,\cdots,N\text{-}1, S_d(x,y,t) = S^t(x,y,d), S_{\max} = \max_{d=0,1,\cdots,N-1} S_d$$

Modified FitzHugh-Nagumo Type Reaction-Diffusion Equations

- The model:

D_u,D_v: Diffusion coefficients
0< ε <<1: Small constant

$$\frac{\partial u}{\partial t} = D_u \nabla^2 u + \frac{1}{\varepsilon}\left[u(1-u)(u-a) - v\right]$$

(*u*:Activator)

$$\frac{\partial v}{\partial t} = \underbrace{D_v \nabla^2 v}_{\text{Additional term}} + u - bv$$

(*v*:Inhibitor)

$D_u<D_v$ => Region Growing
+ Self-Inhibition Mechanism

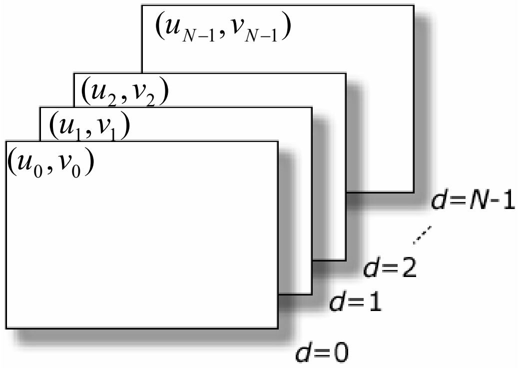
Proposed Model: Multi-Sets of Reaction-Diffusion Equations

- The model

$$\frac{\partial u_d}{\partial t} = D_u \nabla^2 u_d + \frac{1}{\varepsilon}\left[u_d(1-u_d)(u_d - a(u_{\max})) - v_d\right] + rC$$

$$\frac{\partial v_d}{\partial t} = D_v \nabla^2 v_d + u_d - v_d$$

$u_{\max} = \max\{u_0, u_1, \cdots, u_{d-1}, u_{d+1}, \cdots, u_{N-1}\}$ $M(x,y) = \underset{d=0,1,\ldots,N-1}{\arg\max}\, u_d$



- Mutual Inhibition Mechanism

$$a(u_{\max}) = \frac{1}{4}\left[1 + \tanh(u_{\max} - a_0)\right] \times \frac{1}{2}\left[1 + \tanh(|\,d\,|)\right]$$

|d|: Distance between the current disparity level and the level of $u_{\max}$.

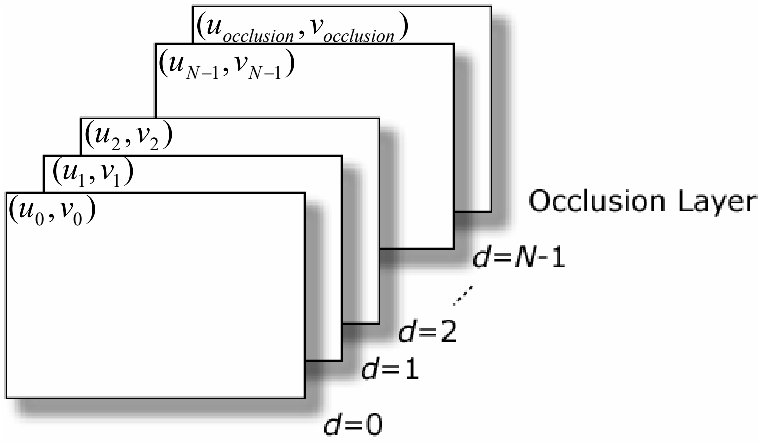
Detection of Occlusion Area

- Prepare an additional disparity layer having two variables ($u_{\text{occlusion}}, v_{\text{occlusion}}$) for detecting occlusion area.
- Correlation map:

$$C_{occlusion} = 1 - \max\{C_{d=0}, C_{d=1}, \cdots, C_{N-1}\}$$

- Disparity map:

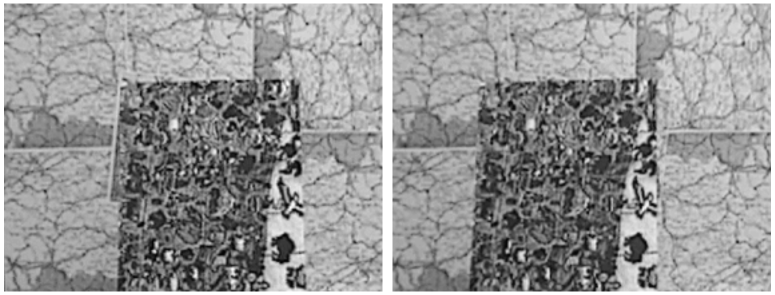
$$M(x,y) = \underset{d=0,1,\ldots,N-1,\text{Occlusion}}{\arg\max} \; u_d$$



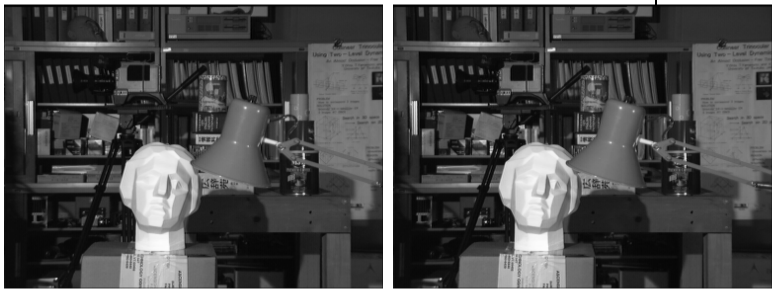
Numerical Computation

- First order approximation for temporal derivatives on u_d and v_d.
- Laplacian operator ∇^2: 5 points approximation with the Crank-Nicolson method.
- Linear algebraic equations are solved by the Gauss-Seidel method.
- Neumann boundary condition.
- Initial conditions: $u_d = v_d = 0$.

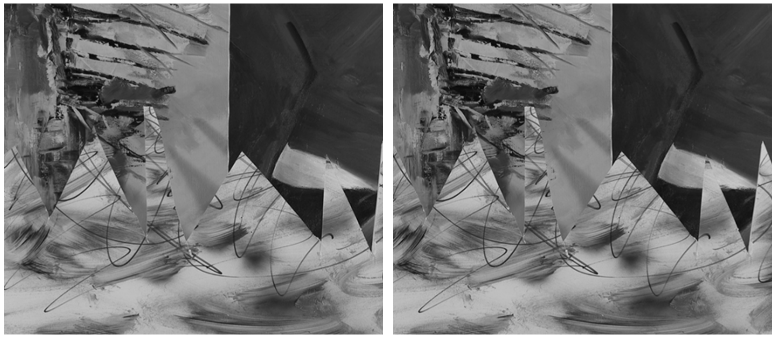
Performance Evaluation:
Test Images Provided by Scharstein



MAP
(384X216 pixel, 30 disparity levels)



TSUKUBA
(384X288 pixel, 16 disparity levels)

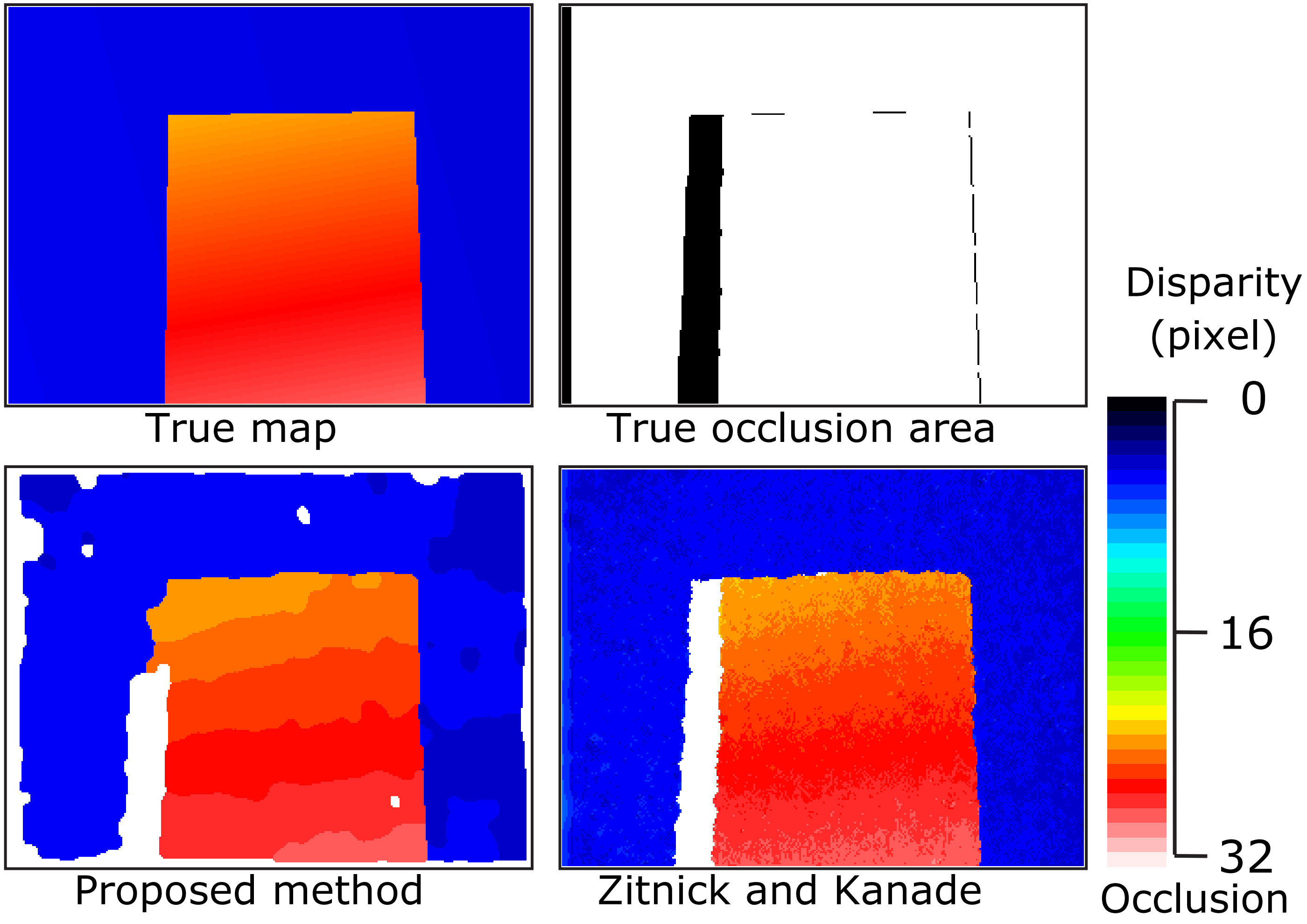


SAWTOOTH
(434X380 pixel, 20 disparity levels)

VENUS
(434X383 pixel, 20 disparity levels)

http://www.middlebury.edu/stereo
Scharstein, D., Szeliski, R.: A taxonomy and evaluation of dense two-frame stereo correspondence algorithms. IJCV **Vol.47**, pp.7-42 (2002)

Performance Evaluation: Results for MAP



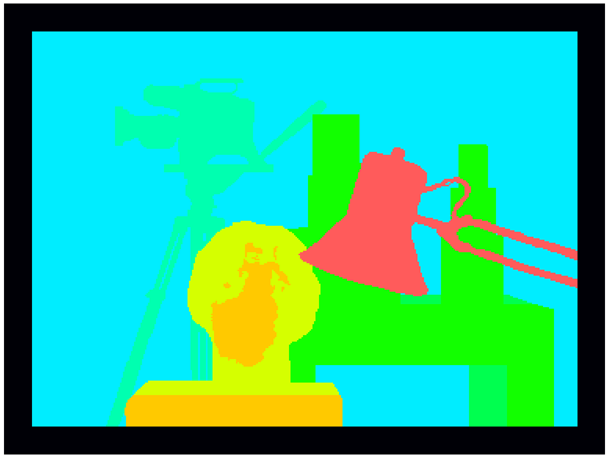
True map

True occlusion area

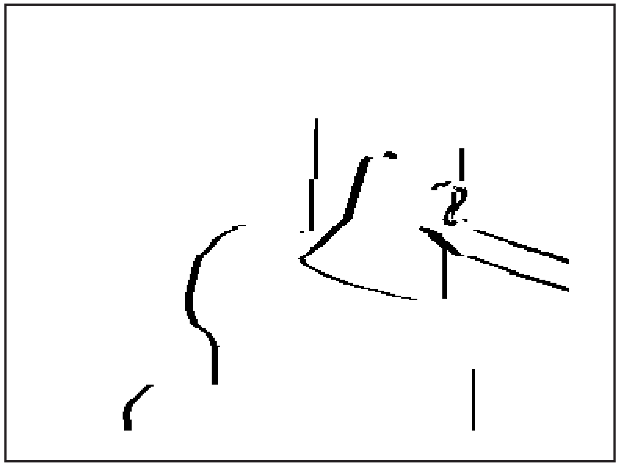
Proposed method

Zitnick and Kanade

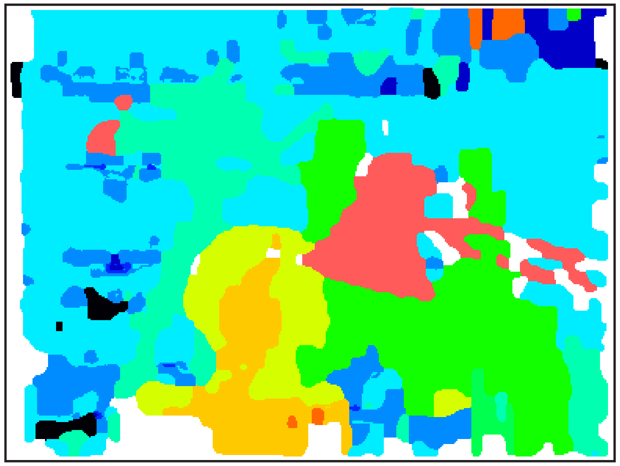
Performance Evaluation: Results for TSUKUBA



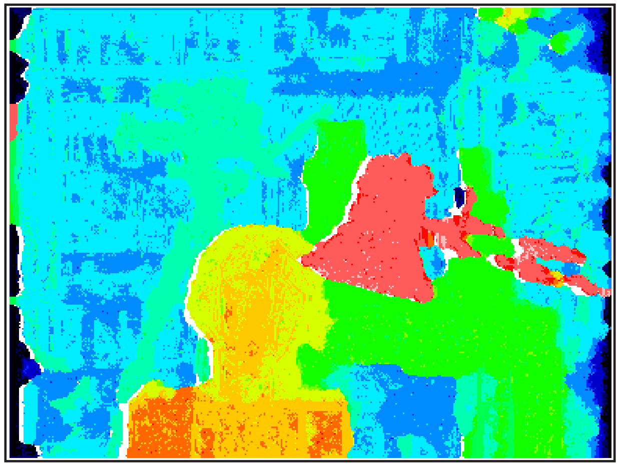
True map



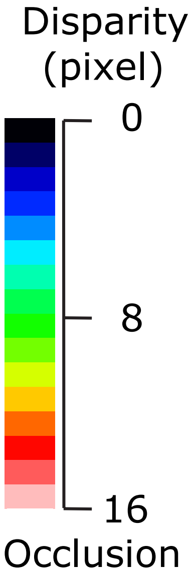
True occlusion area



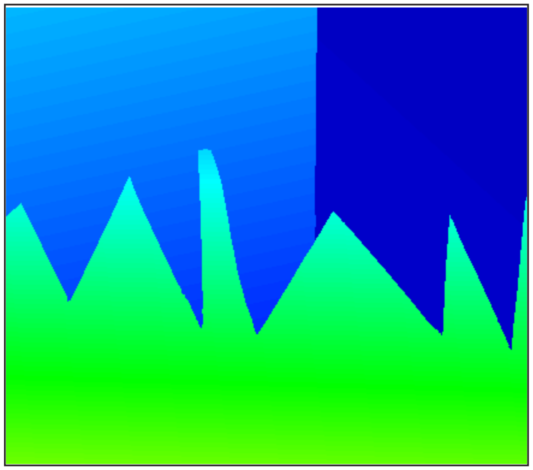
Proposed method



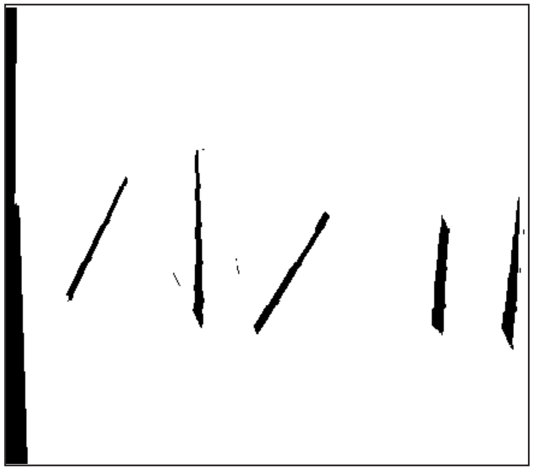
Zitnick and Kanade



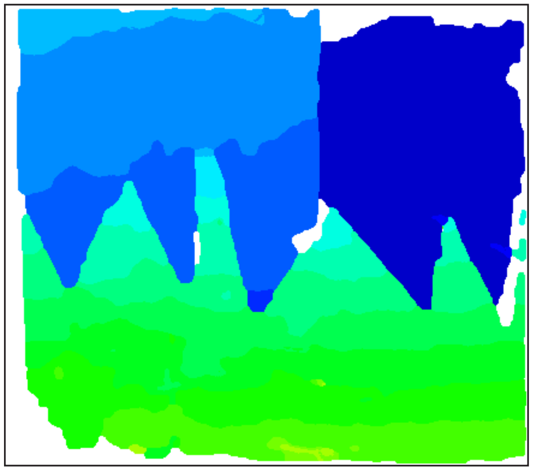
Performance Evaluation: Results for SAWTOOTH



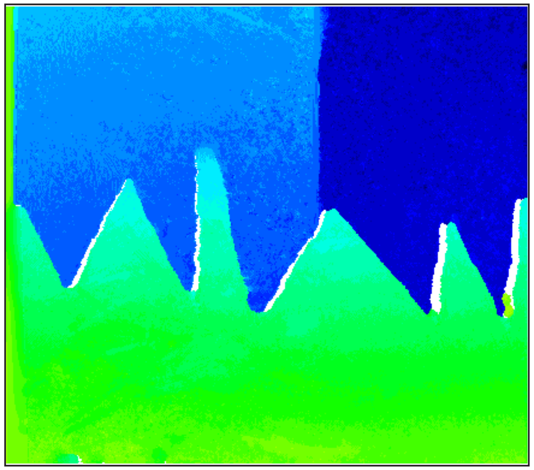
True map



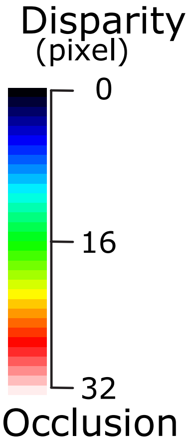
True occlusion area



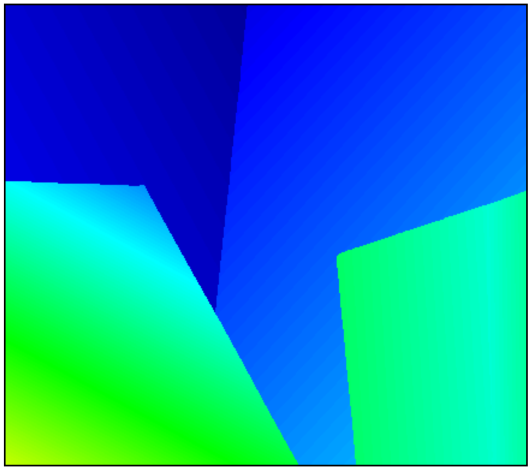
Proposed method



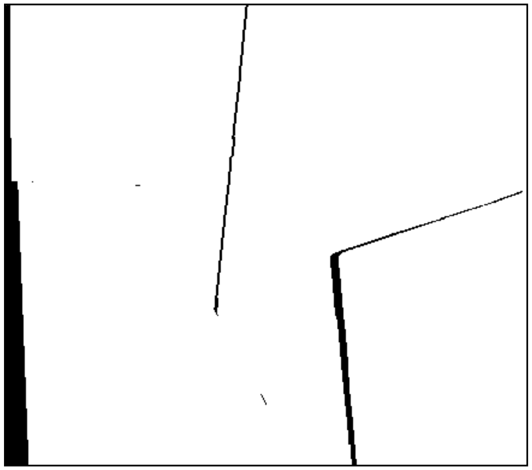
Zitnick and Kanade



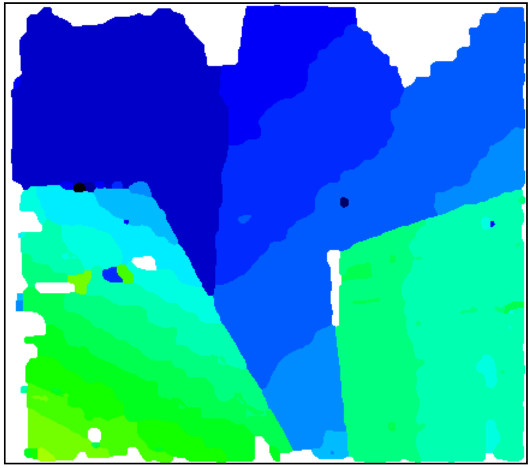
Performance Evaluation: Results for VENUS



True map

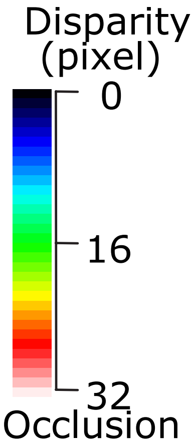


True occlusion area



Proposed method

Zitnick and Kanade



Parameter Values Utilized in Performance Evaluation

- Our proposed model
  - $\delta x = \delta y = 0.5$, $\delta t = 10^{-3}$

    (finite differences for discretization)
  - $D_u = 0.5$, $D_v = 1.0$, $a_0 = 0.15$, $b = 10$, $\varepsilon = 10^{-2}$
  - $r = 3.0$ ($r = 3.6$ for the occlusion layer)

- Cooperative model proposed by Zitnick and Kanade
  - $\alpha = 2.0$, $T = 0.001$, $C = 0.08$ if $C < 0.08$

Performance Evaluation: Error Measures

- Root-Mean-Squares (pixel)

$$R = \left[\frac{1}{N_R} \sum_{(x,y)\in F_{\neg o}} \left\{ M_t(x,y) - M_c(x,y) \right\}^2 \right]^{1/2}$$

N_R: Number of points, $F_{\neg o}$: Domain of non-occlusion area

- Bad-Match Percentage (%)

$$B_F = \frac{1}{N_B} \sum_{(x,y)\in F} \sigma\left(M_t(x,y) - M_c(x,y), \delta d \right)$$

N_B: Number of points, F: Domain, δd: Threshold value

Scharstein, D., Szeliski, R.: A taxonomy and evaluation of dense two-frame stereo correspondence algorithms. IJCV Vol.47, pp.7-42 (2002)

Performance Evaluation: Quantitative Results

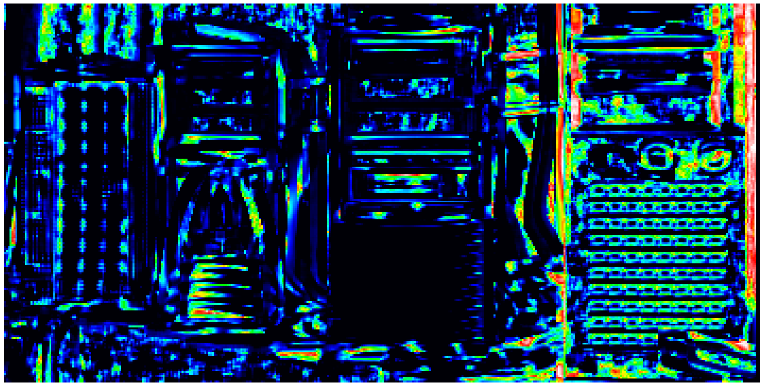
Better performance

Stereo Image	Method	R(pixel)	B_o(%)	$B_{\neg o}(\delta d=1.0)$ (%)	$B_{\neg o}(\delta d=0.5)$ (%)
MAP	ZK	1.210	14.6	1.417	17.5
	PM	1.175	40.7	0.953	12.4
TSUKUBA	ZK	1.057	74.3	3.967	32.2
	PM	1.352	74.8	8.290	27.0
SAWTOOTH	ZK	0.802	59.0	3.604	19.2
	PM	0.807	89.9	3.197	11.0
VENUS	ZK	0.688	72.8	4.639	20.6
	PM	0.758	79.4	5.747	10.2

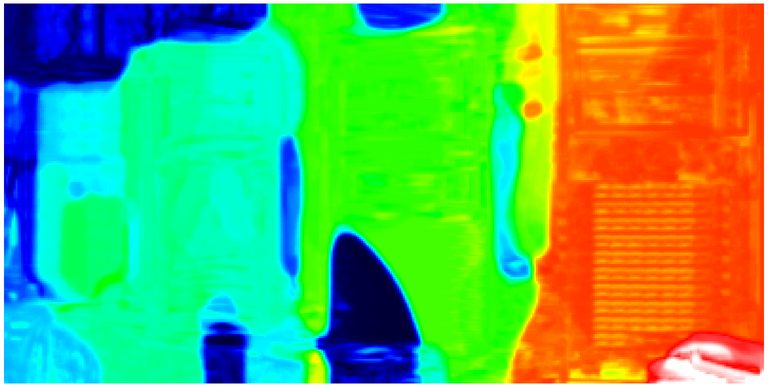
ZK: Cooperative model proposed by Zitnick and Kanade, PM: Our proposed model
B_o: Bad-match percentage for occlusion area, $B_{\neg o}$: Bad-match percentage for non-occlusion area

Experimental Results for Real Stereo Images (1)

400X300
(pixel)



Correlation Map $C(d=43)$

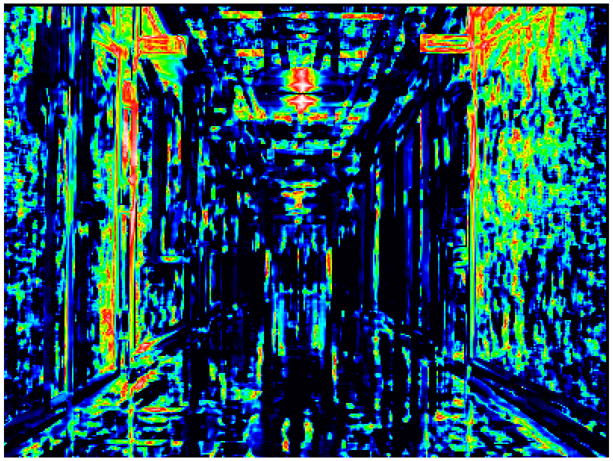


$D_u=1.0$
$D_v=2.0$
$a_0=0.25$
$b=20$
$\varepsilon=10^{-2}$
$r=2.55$

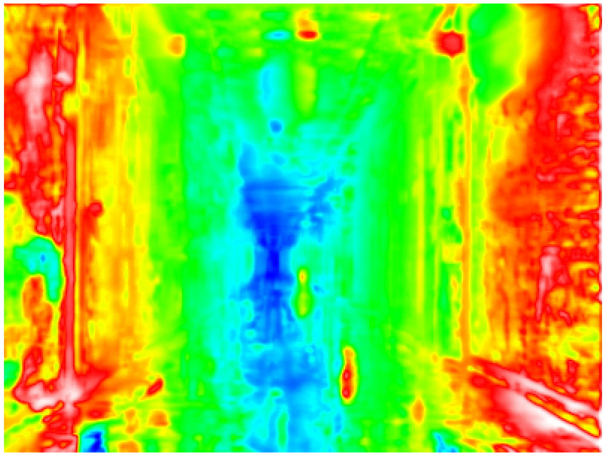
$N=30$(disparity levels)
(We do not utilize the algorithm for detection of occlusion area.)

Experimental Results for Real Stereo Images (2)

400X300 (pixel)



Correlation Map $C(d=35)$



$D_u=1.0$
$D_v=2.0$
$a_0=0.20$
$b=10$
$\varepsilon=3X10^{-2}$
$r=2.55$

N=46(disparity levels)
(We do not utilize the algorithm for detection of occlusion area.)