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Long-Range Inhibition in Reaction-Diffusion Algorithms Designed for Edge Detection and Stereo Disparity Detection

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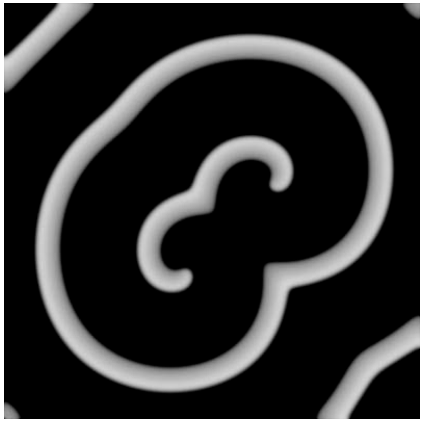
Outline

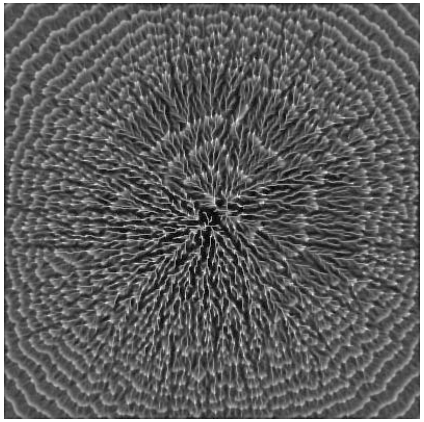
- Introduction
- Motivation
- Reaction-diffusion algorithms
  - Edge detection
  - Stereo disparity detection
  - Experimental results
- Proposal on the stereo algorithm
  - Anisotropic strong inhibitory diffusion
- Conclusion

Introduction:
Reaction-Diffusion Systems in Pattern Formation Processes

- Chemical reaction system
  - Belousov-Zhabotinsky (BZ) reaction
- Biological system
  - Amoeba: Dictyostelium discoideum
- Self-organized dynamic patterns



Numerical simulation of
Belousov-Zhabotinsky reaction
Keener & Tyson, *Physica D*, 1986



Numerical simulation of
Dictyostelium discoideum
Höfer et al., *Physica D*, 1995

A Reaction-Diffusion System

- Diffusion equation with a source term

$$\frac{\partial u}{\partial t} = D_u \nabla^2 u + S$$

D_u: diffusion coefficient
S: source term

- General form of a reaction-diffusion system

$$\frac{\partial u}{\partial t} = D_u \nabla^2 u + f(u,v)$$

$$\frac{\partial v}{\partial t} = \underbrace{D_v \nabla^2 v}_{\text{diffusion terms}} + \underbrace{g(u,v)}_{\text{reaction terms}}$$

D_u, D_v : diffusion coefficients

Introduction:

Strong Inhibitory Diffusion in Biological Systems

- Turing scenario in biological systems
  - Turing, *Phil. Trans. Roy. Soc.*, 1952
    - Strong inhibitory diffusion induces spatial periodic waves.
  - Gierer & Meinhardt, *Kybernetik*, 1972
    - Realistic models of biological pattern formation
  - Kondo & Asai, *Nature*, 1995
    - Pattern on fish skin is caused by the Turing scenario
- Key point: Strong inhibitory diffusion

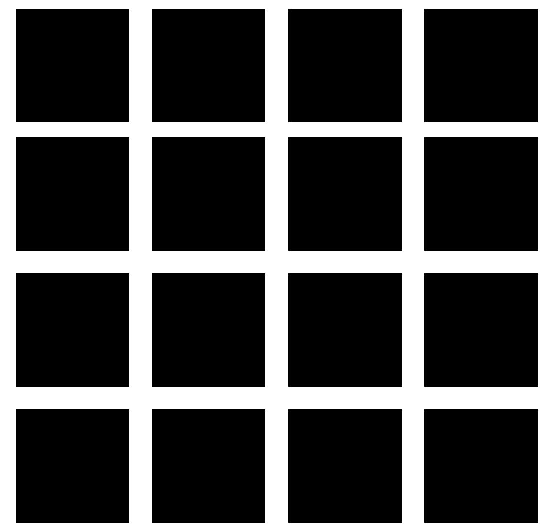
$\Leftrightarrow$ Long-range inhibition?

Does the Turing pattern exist
in biological systems ?

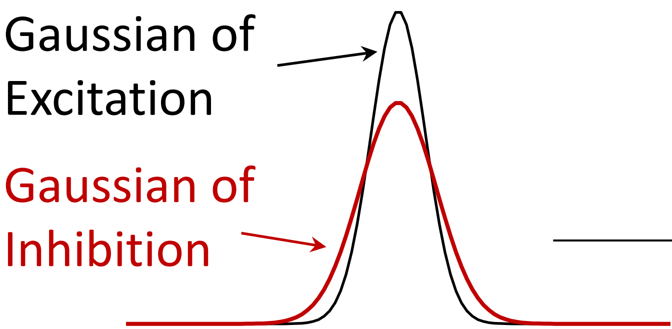
Long-Range Inhibition in Biological Vision

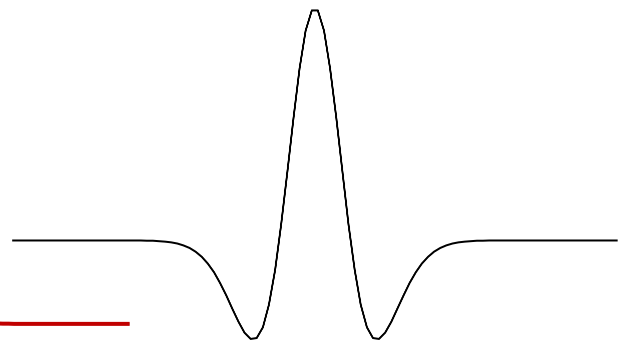
- DoG filter
  - Marr & Hildreth, *Proc. Roy. Soc. Lond.*, 1980
- Mach bands effect
  - human visual system
    - Mach, *Akademie der Wissenschaften*, 1865
  - lateral eyes of Limulus (crab)
    - Ratliff & Hartline, *J. Gen. Physiol.*, 1959

Mach bands



Is the Hermann grid illusion due to long-range inhibition?





DoG:
difference of two Gaussians

Motivation

- Image processing with a reaction-diffusion system (photo-sensitive chemical reaction)
  - Kuhnert et al., *Nature*, 1989

- Long-range inhibition in nature and in visual systems.

$\Rightarrow$ We confirm how long-range inhibition or rapid inhibitory diffusion works in reaction-diffusion algorithms in edge detection and stereo disparity detection.

The FitzHugh-Nagumo Reaction System

- Single reaction system (ODE)

$$\begin{cases} \dfrac{\mathrm{d}u}{\mathrm{d}t} = \dfrac{1}{\varepsilon}\left[u(u-a)(1-u)-v\right] \\ \dfrac{\mathrm{d}v}{\mathrm{d}t} = u - bv \end{cases}$$

u: activator
v: inhibitor

a,b,ε : constants

$0<\varepsilon<<1$

- Reaction-diffusion system (PDE)

$$\begin{cases} \dfrac{\partial u}{\partial t} = D_u \nabla^2 u + \dfrac{1}{\varepsilon}\left[u(u-a)(1-u)-v\right] \\ \dfrac{\partial v}{\partial t} = D_v \nabla^2 v + u - bv \end{cases}$$

Time-evolving equations starting from initial states of $u(x,y,t=0)$ and $v(x,y,t=0)$.

System Behaviour of the FitzHugh-Nagumo ODE System

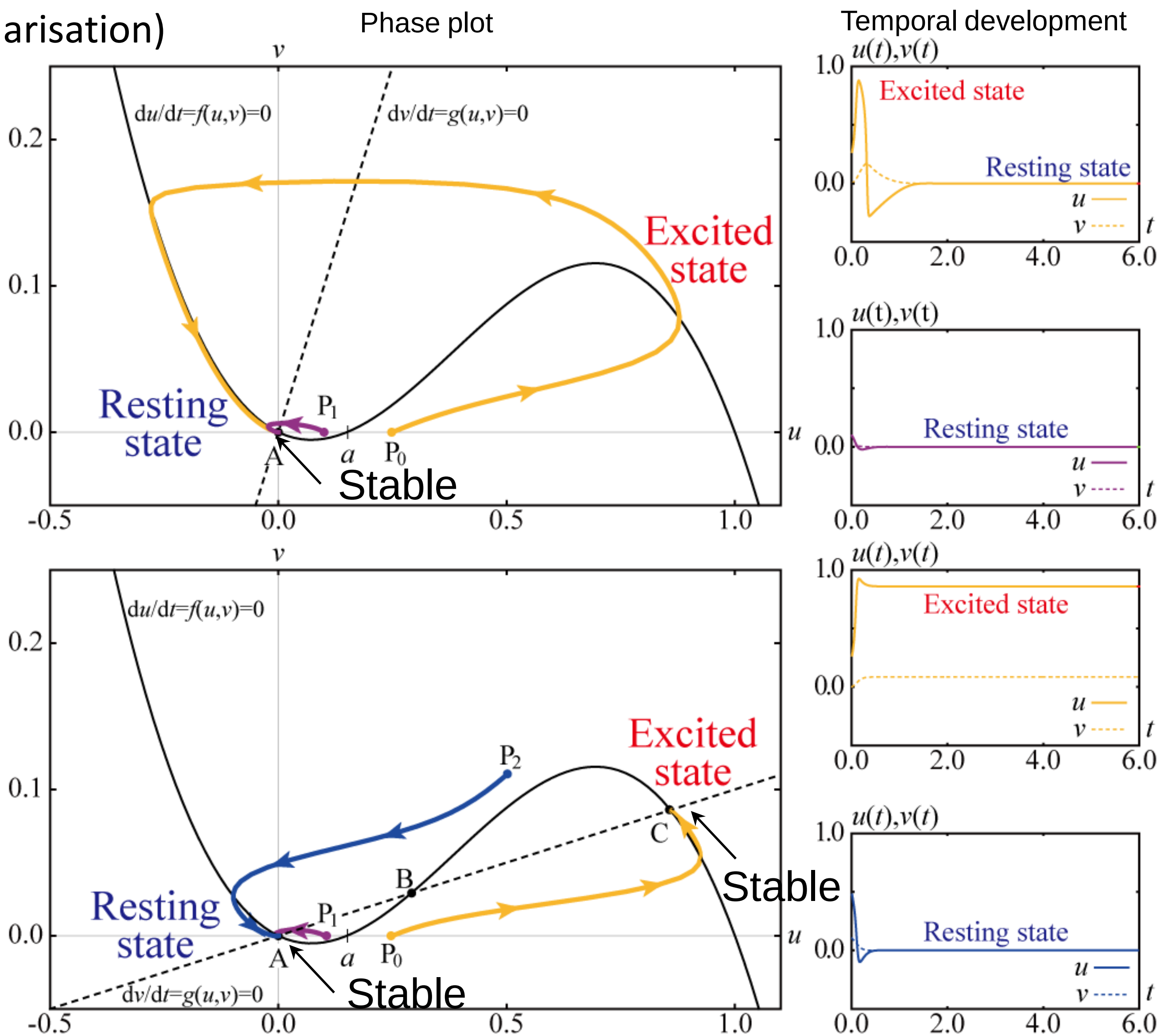
(threshold level *a* for binarisation)

(a) Uni-stable system

$$\frac{du}{dt} = f(u,v) = \frac{1}{\varepsilon}\left[u(u-a)(1-u)-v\right]$$

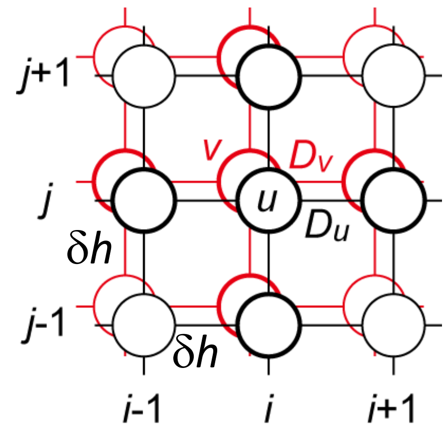
$$\frac{dv}{dt} = g(u,v) = u - bv$$

(b) Bi-stable system



Spatially Coupled Uni-stable Reaction Systems

- Uni-stable ODE systems placed at image grids
  - Nomura et al., *J. Phys. Soc. Jpn.*, 2003
  - Kurata et al., *Phys. Rev. E*, 2009



$$\frac{\mathrm{d}u_{i,j}}{\mathrm{d}t} = \frac{D_u}{\delta h^2}\left[\overline{u_{i,j}} - 4u_{i,j}\right] + \frac{1}{\varepsilon}\left[u_{i,j}(u_{i,j}-a)(1-u_{i,j}) - v_{i,j}\right]$$

$$\frac{\mathrm{d}v_{i,j}}{\mathrm{d}t} = \frac{D_v}{\delta h^2}\underbrace{\left[\overline{v_{i,j}} - 4v_{i,j}\right]}_{\text{Spatial coupling}} + u_{i,j} - bv_{i,j}$$

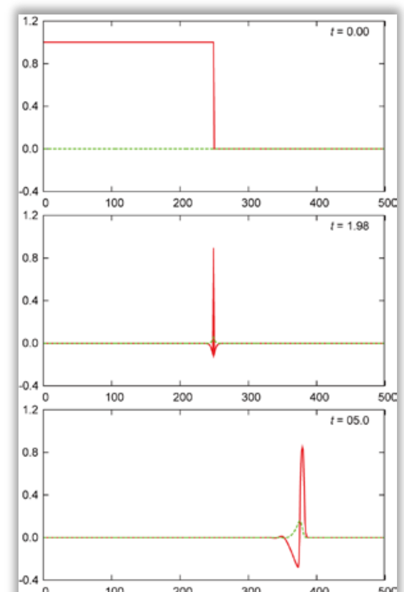
$\overline{u_{i,j}}$: average in the nearest four points.

- Strong inhibition: D_u<<D_v

  ⇒ Stationary pulse at an edge position

- Weak inhibition: D_u>D_v

  ⇒ Propagating pulse in space



Initial states
u and v

u and v
(D_u<<D_v)

u and v
(D_u > D_v)

Examples for a Real Image

- Initial states:
  - $u_{i,j}(0)$="image intensity distribution" and $v_{i,j}(0)=0.0$ for any i,j.
- Pulses are self-organised at edges.
  - $D_u \ll D_v$: pulses are stationary at the edges.
  - $D_u > D_v$: pulses propagates and develop as spiral waves.

Real image (512×512 pixels)
Initial state of $u_{i,j}(t=0)$.

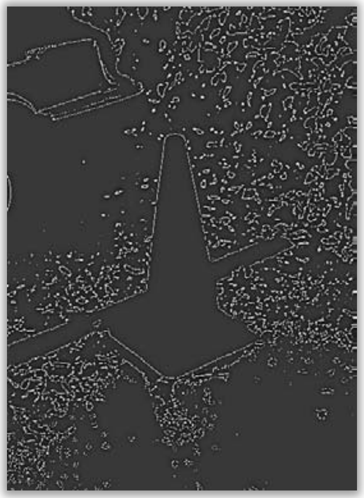
$u_{i,j}(t=10)$ with
$D_v=0.0$ and $\delta h=1/5$

$u_{i,j}(t=10)$
with $D_v=5.0$ and $\delta h=1/2$

Courtesy of Heath et al.: "Edge detector comparison",
http://marathon.csee.usf.edu/edge/edge_detection.html

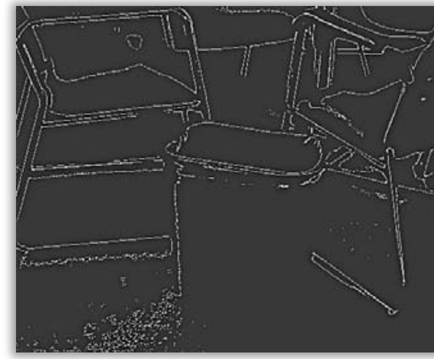
Other parameter settings:
$D_u=1.0$, $a=0.05$, $b=1.0$, $\varepsilon=1.0\times10^{-2}$, $\delta t=1/10000$

Examples for Real Images



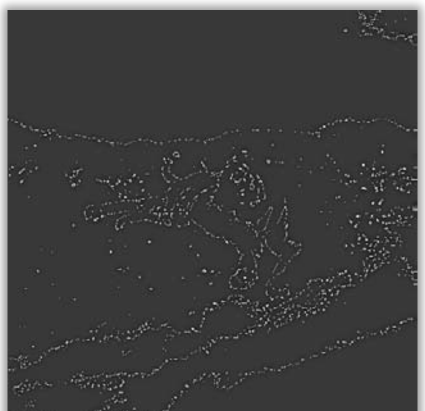
437x604 (pixels)

a=0.20



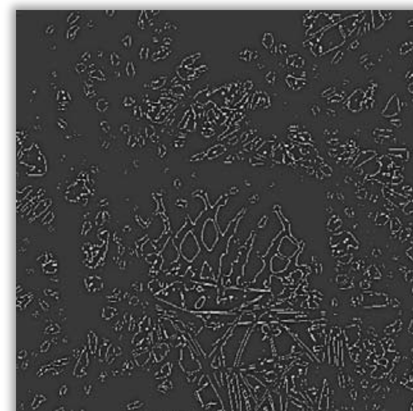
437x604 (pixels)

a=0.20



512x512 (pixels)

a=0.30



512x512 (pixels)

a=0.20

Courtesy of Heath et al.: “Edge detector comparison”, http://marathon.csee.usf.edu/edge/edge_detection.html

Other parameter settings:
D_u=1.0, D_v=5.0, b=1.0, ε=1.0 x10^{-3}, δh=1/2, δt=1/1000

Reaction-Diffusion Stereo Algorithm

- Multi-layered network of bi-stable reaction-diffusion systems
  - Nomura et al., *Mach. Vis. Appl.*, 2009

Reaction Diffusion Eqs.

$$\begin{cases} \dfrac{\partial u_n}{\partial t} = D_u \nabla^2 u_n + f(u_n, v_n, u_{\max}) + \mu C(x, y, d_n) \\ \dfrac{\partial v_n}{\partial t} = D_v \nabla^2 v_n + g(u_n, v_n) \end{cases}$$

$$u_{\max} = \max_{n' \in \Theta} u_{n'}$$

Θ: inhibition area

Reaction Terms & Threshold $a(.)$

$$\begin{cases} f(u_n, v_n, u_{\max}) = \dfrac{1}{\varepsilon}\left[u_n (u_n - a(u_{\max}))(1 - u_n) - v_n\right] \\ g(u_n, v_n) = u_n - b v_n \end{cases}$$

$$a(u_{\max}) = a_0 + \frac{u_{\max}}{2}\left[1 + \tanh\left(\left|d_n - \arg\max_{n'} u_{n'}\right| - a_1\right)\right]$$

- Disparity map:

$$M(x, y, t) = \arg\max_{n \in \{0, 1, \cdots, N-1\}} u_n(x, y, t)$$

μ: constant
N: total number of disparity levels
d_n: disparity level at the nth network
$C(x,y,d_n)$: matching cost function

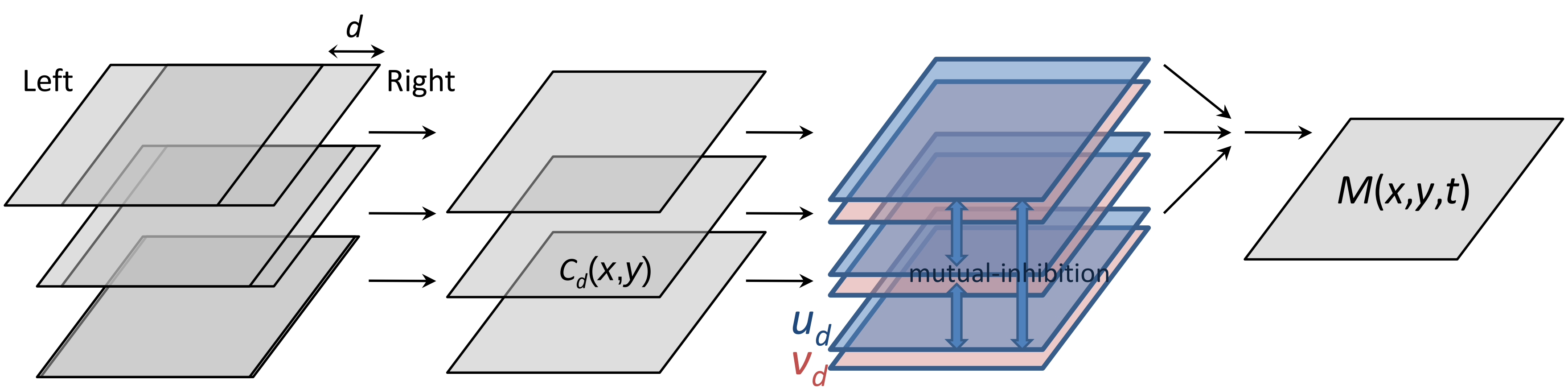
Diagram of the Stereo Algorithm

Reaction-diffusion systems:

$$\begin{cases} \dfrac{\partial u_d}{\partial t} = D_u \nabla^2 u_d + f(u_d, v_d, u_{\max}) + \mu C_d \\ \dfrac{\partial v_d}{\partial t} = D_v \nabla^2 v_d + g(u_d, v_d) \end{cases}$$

- d: disparity level
- C_d: matching cost function

Disparity map: $$M(x, y, t) = \arg\max_d u_d(x, y, t)$$



A pair of stereo images overlapped with several possible disparity levels d.

Correlation maps computed for overlapped stereo images.

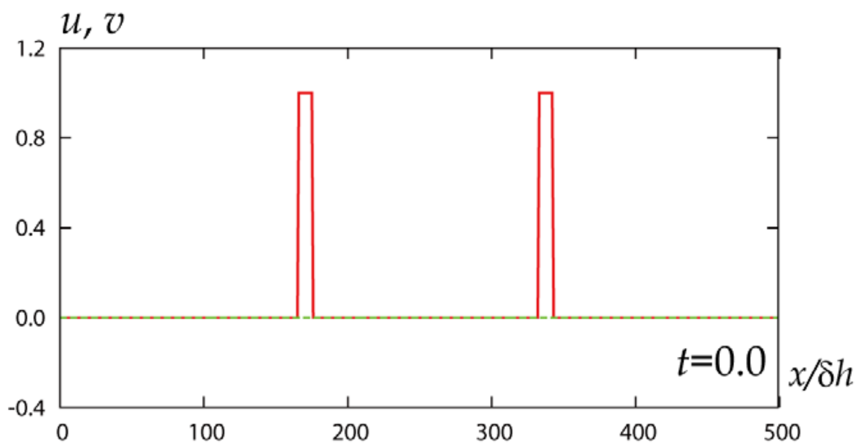
A multi-layer network of reaction-diffusion systems mutually connected via $a(u_{max})$.

A stereo disparity map obtained with the argmax u_d.

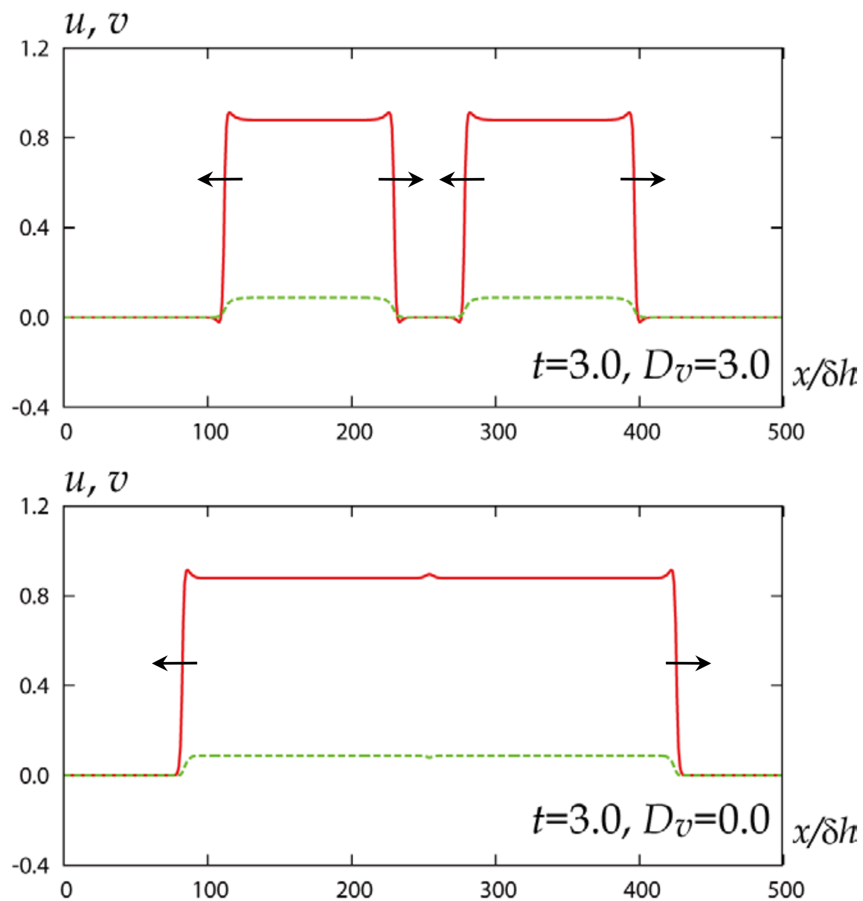
Reaction-diffusion algorithm: stereo disparity detection

Bi-stable FitzHugh-Nagumo Reaction-Diffusion System

- Propagating waves realise a filling-in process.
- Wave speed depends on D_v.



Initial states of *u* and *v*.



Parameter settings:
D_u=1.0, D_v=3.0 or D_v=0.0
a=0.05, b=10.0, ε=1/100

Reaction-diffusion algorithm: stereo disparity detection

Experimental Results for Middlebury Stereo Vision Data

- The Middlebury stereo vision page provides
  - stereo image pairs,
  - ground-truth data of disparity maps,
  - definition of areas (occlusion area & depth discontinuity area),
  - scores of other stereo algorithms
- Example of stereo image pairs (left reference image)

(a) CONES 450X375 pixels
60 disparity levels

(b) TEDDY 450X375 pixels
60 disparity levels

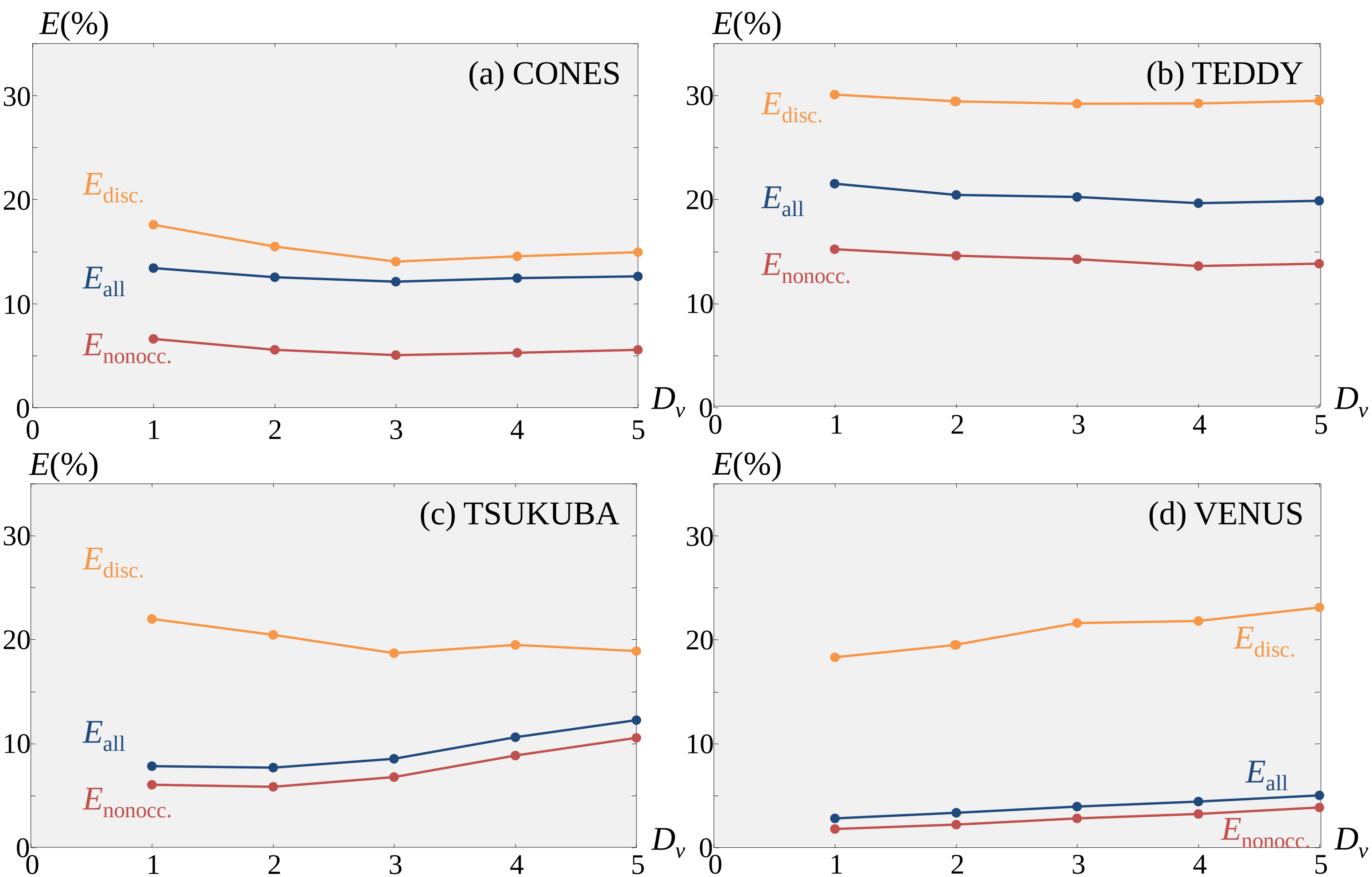
(c) TSUKUBA
384X288 pixels
15 disparity levels

(d) VENUS 434X383 pixels
30 disparity levels

Courtesy of Scharstein & Szeliski: "Middlebury Stereo Vision Page", http://vision.middlebury.edu/stereo/

Reaction-diffusion algorithm: stereo disparity detection

Dependence of Inhibitory Diffusion on Performance



Parameter settings:

$D_u = 1.0$

$a_0 = 0.13$

$a_1 = 1.5$

$b = 10$

$\varepsilon = 1.0 \times 10^{-2}$

$\mu = 3.0$

$\delta h = 1/5$

$\delta t = 1/100$

E (%): Bad-Match-Percentage error measure

nonocc.: non-occlusion area
all: all area
disc.: depth discontinuity area

Reaction-diffusion algorithm: stereo disparity detection

Anisotropic Inhibitory Diffusion in the Stereo Algorithm

- Imposing strong inhibitory diffusion $D_v(x,y)$ at around depth-discontinuity areas.

$$\begin{cases} \dfrac{\partial u_n}{\partial t} = D_u \nabla^2 u_n + f(u_n, v_n, u_{\max}) + \mu C(x, y, d_n) \\ \dfrac{\partial v_n}{\partial t} = \nabla \cdot \boxed{\left[D_v(x, y) \nabla v_n \right]} + g(u_n, v_n) \end{cases}$$

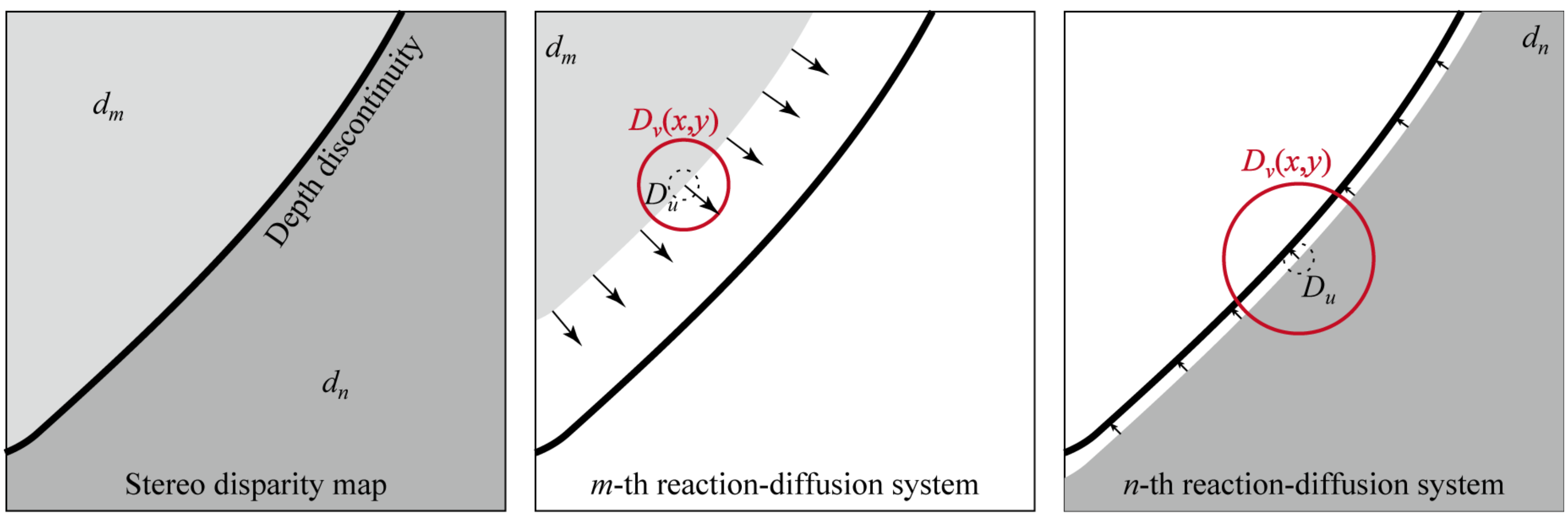
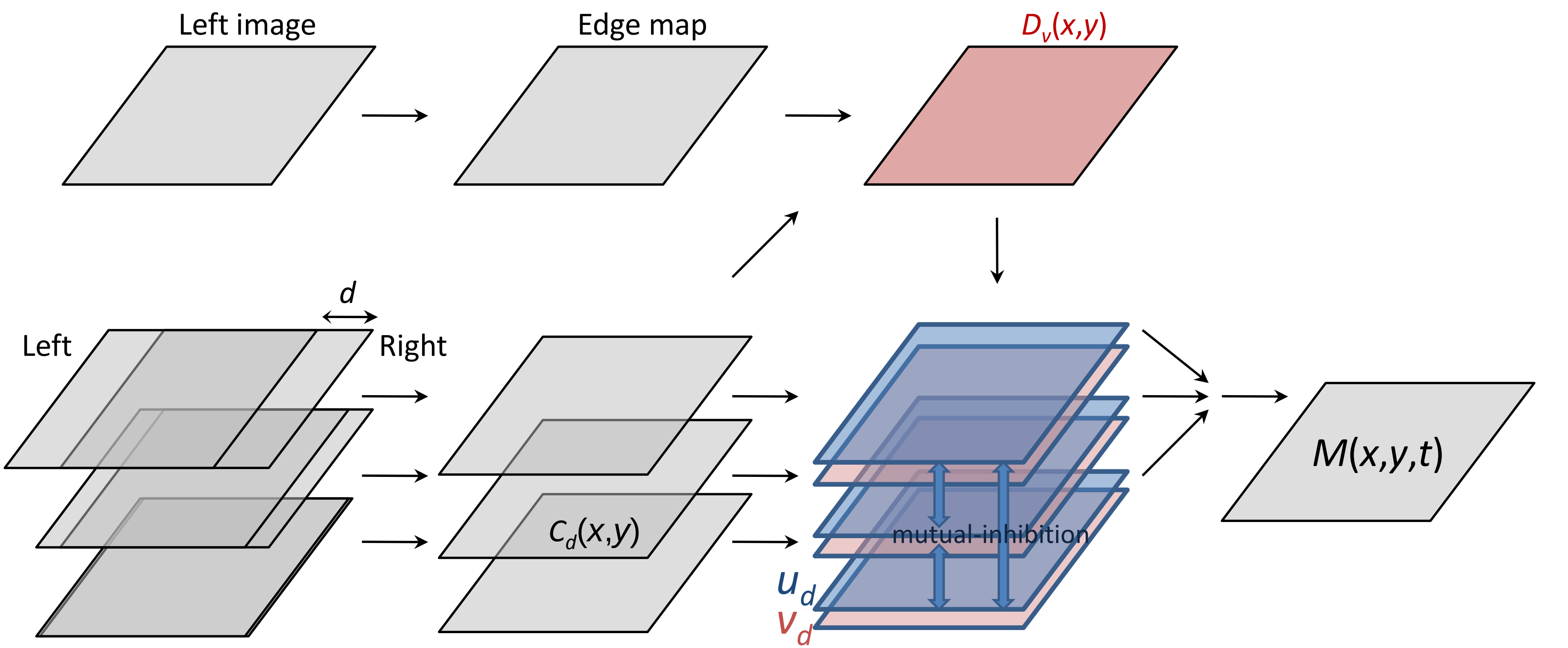


Diagram of the Proposed Stereo Algorithm

Edge map is computed with a reaction-diffusion system.

Estimate Dv(x,y) according to the edge map and an initial disparity map estimated from the correlation maps.



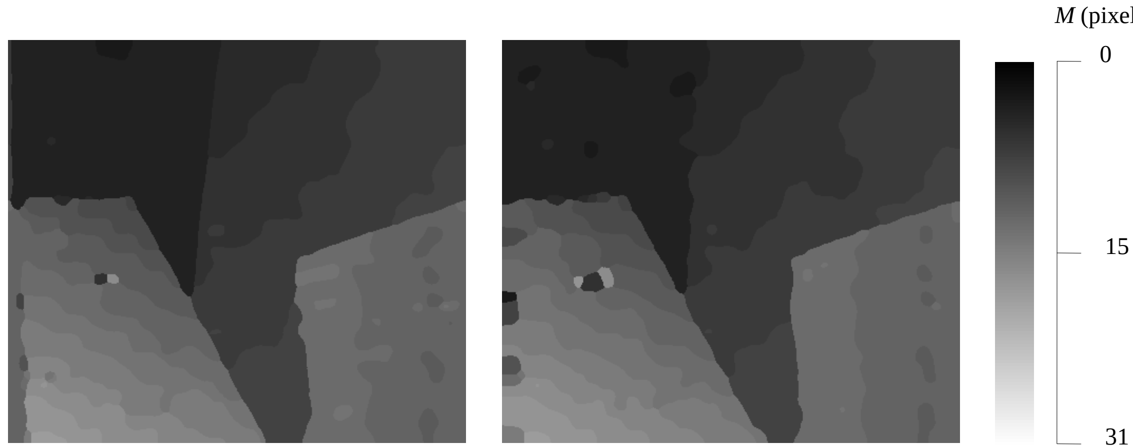
A pair of stereo images overlapped with several possible disparity levels d.

Correlation maps computed for overlapped stereo images.

A multi-layer network of reaction-diffusion systems mutually connected via $a(u_{max})$.

A stereo disparity map obtained with the argmax u_d.

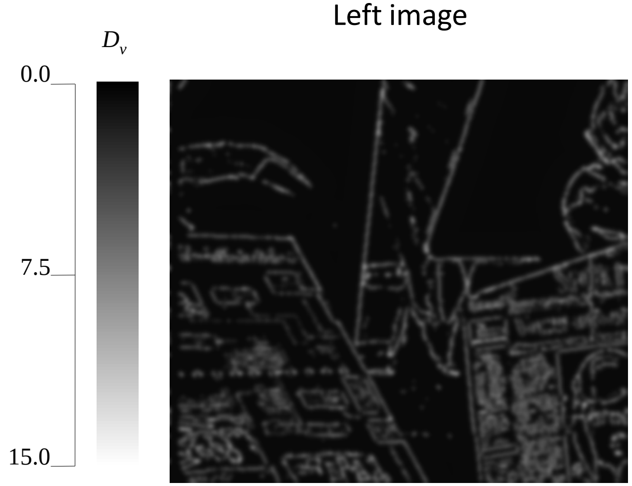
Results on the Stereo Image Pair VENUS



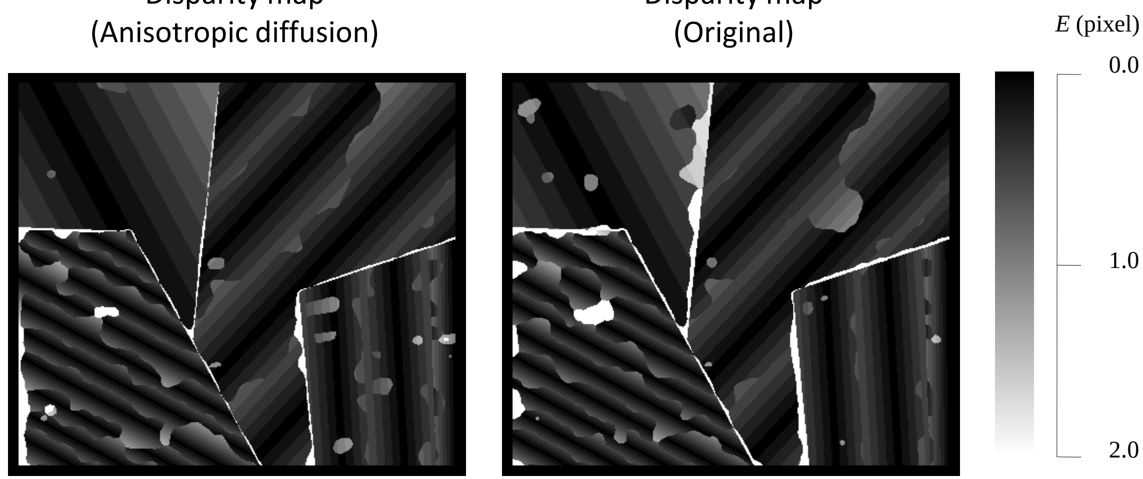
Left image

Disparity map (Anisotropic diffusion)

Disparity map (Original)



D_v



Error distribution (Anisotropic diffusion)

Error distribution (Original)

Conclusion

- Reaction-diffusion algorithms for edge detection and stereo disparity detection.
- Strong inhibition works for stable results in edge detection.
- Strong inhibitory diffusion brings better result in depth discontinuity areas in stereo disparity detection.

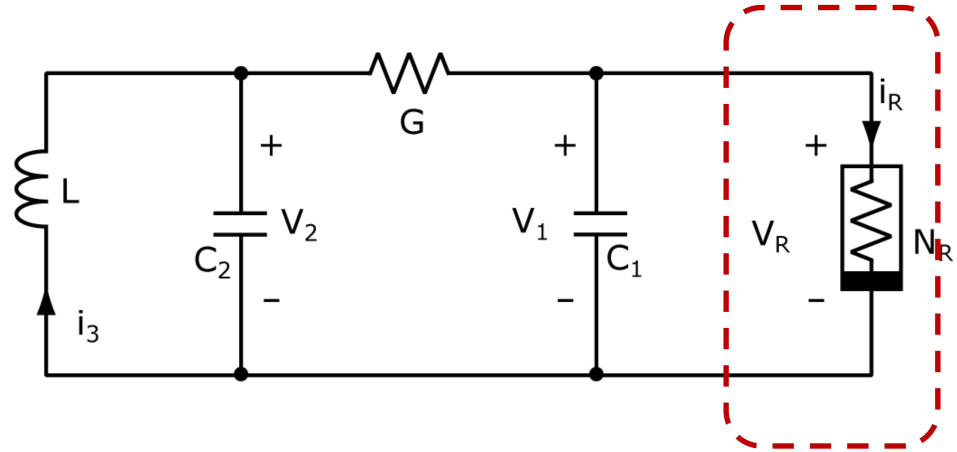
Acknowledgment:
The present study was supported in part by the Grant-in-Aid for Scientific Research (C) (No. 20500206) from the Japan Society for the Promotion of Science.

Thank you for your attention!

Appendix: Reaction-Diffusion System with Chua's Circuit

- Ordinary Differential Equation (ODE) System:
  - Matsumoto, *IEEE-CS*, 1984
  - Chua et al., *IEEE-CS*, 1995

$$\begin{cases} C_1 \dfrac{\mathrm{d}v_1}{\mathrm{d}t} = G(v_2 - v_1) - g(v_1) \\ C_2 \dfrac{\mathrm{d}v_2}{\mathrm{d}t} = G(v_1 - v_2) + i_3 \\ L \dfrac{\mathrm{d}i_3}{\mathrm{d}t} = -v_2 \end{cases}$$



- Spatially coupled Chua's circuits realise a reaction-diffusion system.

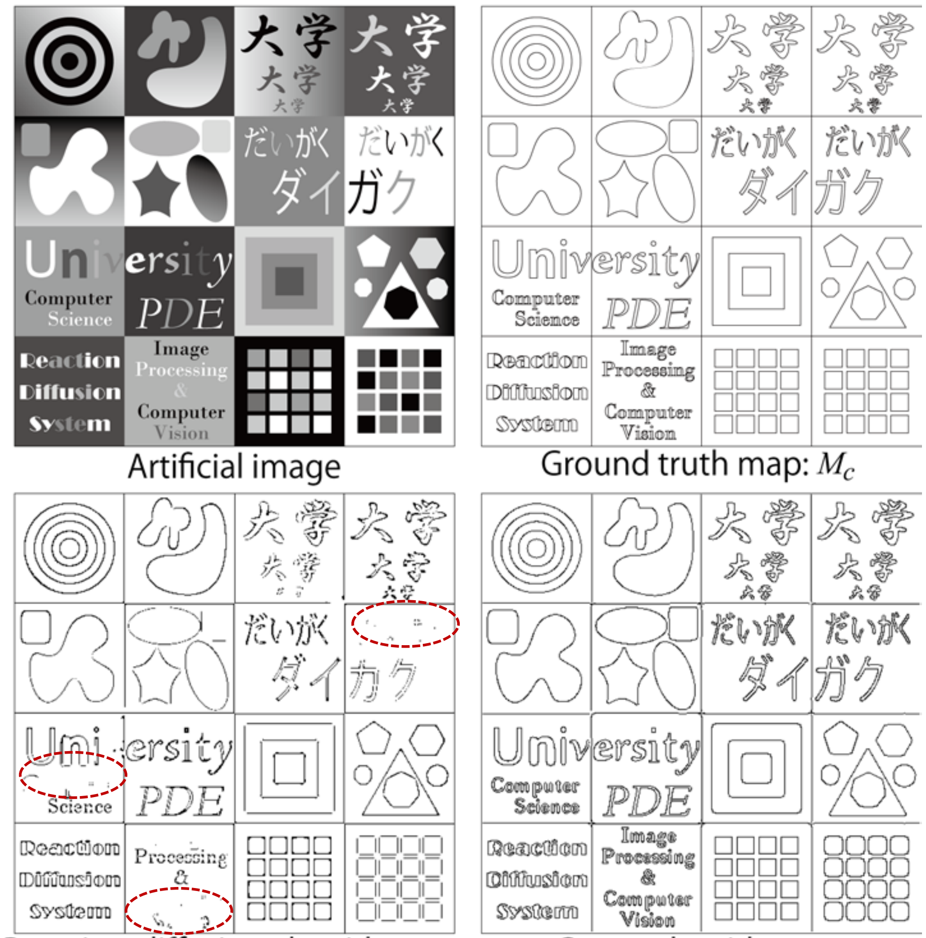
Appendix:
Performance Evaluation of Edge Detection Algorithms

- Tested algorithms:
  - Reaction-diffusion algorithm
  - Canny algorithm
- Error measures:

$$E_c = \frac{1}{|M_c|} \left| M_c \cap \overline{M_o} \right| \times 100\ (\%)$$

$$E_o = \frac{1}{|M_o|} \left| M_o \cap \overline{M_c} \right| \times 100\ (\%)$$

	E_c (%)	E_o (%)
Reaction-Diffusion algorithm	23.07	11.44
Canny algorithm	8.20	8.08



Artificial image

Ground truth map: M_c

Reaction-diffusion algorithm: M_o

Canny algorithm: M_o

Appendix:
Performance Evaluation with BMP and RMS error measures

Image pair	Area	Original (Nomura et al., *Mach. Vis. Appl.*, 2009) BMP (%)	Original RMS (pixel)	IIEI (Nomura et al., *Proc. VISAPP*, 2009) BMP (%)	IIEI RMS (pixel)	Anisotropic inhibitory diffusion BMP (%)	Anisotropic inhibitory diffusion RMS (pixel)	Adapting BP (Klaus et al., *Proc. ICPR*, 2006) BMP (%)
		Reaction-diffusion stereo algorithm						
TSUKUBA	nonocc.	6.77	1.42	8.51	1.54	7.02	1.47	1.11
	all	8.53	1.61	10.23	1.72	8.54	1.64	1.37
	disc.	18.68	2.47	19.42	2.52	18.55	2.60	5.79
VENUS	nonocc.	2.81	0.75	3.17	0.77	1.21	0.59	0.10
	all	3.97	0.92	4.33	0.92	2.44	0.80	0.21
	disc.	**21.64**	**2.01**	**19.62**	**1.88**	**8.12**	**1.52**	1.44
TEDDY	nonocc.	14.26	2.19	14.00	2.38	14.56	2.29	4.22
	all	20.26	3.23	20.00	4.36	20.64	3.32	7.06
	disc.	29.19	3.36	28.89	3.48	27.98	3.45	11.8
CONES	nonocc.	5.03	1.94	5.08	1.85	5.21	1.88	2.48
	all	12.13	3.08	12.35	5.45	13.34	3.15	7.92
	disc.	14.06	3.34	13.67	3.03	14.08	3.17	7.32

nonocc.: non-occlusion area, all: all area, disc.: depth discontinuity area
BMP: Bad-Match-Percentage, RMS: Root Mean Squares

Courtesy of Scharstein & Szeliski: "Middlebury Stereo Vision Page", http://vision.middlebury.edu/stereo/