EDGE DETECTION WITH REACTION-DIFFUSION EQUATIONS HAVING LOCAL AVERAGE THRESHOLD

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The present paper proposes an edge detection algorithm with a modified version of the FitzHugh-Nagumo reaction-diffusion equations. The original FitzHugh-Nagumo reaction-diffusion equations in a mono-stable system have the function of detecting edges from binary images. That is, the equations organize a static impulse at the position being across a fixed threshold value in image brightness distribution; the position of the impulse denotes an edge position. The proposed algorithm detects edges from grey level images. The algorithm computes local average of image brightness distribution with a simple diffusion equation; the local average modulates the threshold value of the reaction-diffusion equations. We apply the previous algorithms and the proposed one to real images and confirm the validity of the proposed algorithm.

Introduction

In image processing, an edge denotes the point having a spatial large gradient in image brightness distribution. A previous edge detection algorithm detects edges by finding zero-crossings from the output of the Laplacian of a Gaussian filter applied to image brightness distribution [1]. The alternative algorithm performing edge detection is available with the difference of two Gaussian filters (DOG) having two different space constants [1].

Two diffusion equations can substitute for two Gaussian filters utilized in the edge detection algorithm. This is due to the fact that the Gaussian filter is the solution of the diffusion equation [2]. The approach utilizing the diffusion equation instead of the Gaussian filter brings a variety of algorithms in image processing. In particular, the introduction of anisotropic diffusion or non-linear diffusion brought further developments of edge detection algorithms [3].

Kuhnert et al. found that a reaction-diffusion system can detect edges with a scientific phenomenon such as a photo-sensitive chemical reaction system [4].

The previous algorithm utilizing the difference of two diffusion equations and that utilising the reaction-diffusion system have an important issue on how to estimate stopping time for an optimal result of edge detection [5]. As time proceeds, edge patterns detected by the two diffusion equations evolve; thus, we need to stop the computation of the diffusion equations for obtaining an optimal result of edge detection. Edges detected by the reaction-diffusion system appear as a transient phenomenon; thus, they disappear after a finite duration of time.

The authors previously proposed an edge detection algorithm [6] utilizing the FitzHugh-Nagumo reaction-diffusion equations [7,8], which imitate the system behaviour of the reaction-diffusion system found by Kuhnert et al. The algorithm imposed a certain condition on the ratio of the two diffusion coefficients of the FitzHugh-Nagumo reaction-diffusion equations. The condition helps to obtain static patterns of edges. Thus, it is unnecessary to estimate stopping time in the edge detection algorithm. In addition, the condition helps to preserve sharp corners and feature points contained in an original image.

However, another important issue on edge detection is still remaining in the previous algorithm proposed by the authors. That is, the FitzHugh-Nagumo reaction-diffusion equations detect edges for binary image; edges detected by the equations do not necessarily correspond to those generally defined for grey level image.

The present paper proposes an improved edge detection algorithm on the previous algorithm proposed by the authors. The algorithm detects edges having large spatial gradients in grey level image. The improvement of the algorithm is due to the fact that original brightness level intersects with its spatial local average level at the edge point. We confirm the validity of the proposed algorithm through experiments for real images.

Previous edge detection algorithms

An edge detection algorithm proposed by Marr and Hildreth detects an edge point by finding a zero-crossing point in the output of the Laplacian of a Gaussian filter (see Fig.1) [1]. An alternative edge detection algorithm utilizes two Gaussian filters with two different space constants. Since the Gaussian filter is the solution of a simple diffusion equation, the algorithm can be re-organized with two diffusion equations as follows [2]:

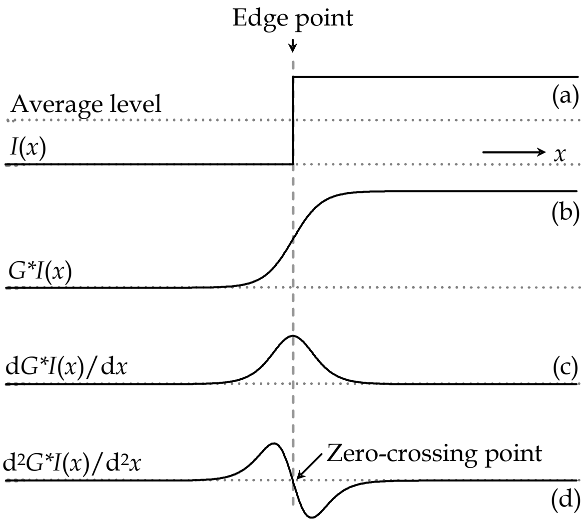

Fig. 1. Definition of an edge point and its detection with the Laplacian of a Gaussian filter. (a) Original image brightness distribution function $I(x)$ in one-dimensional space x; (b) Gaussian filter output $G*I(x)$; (c) first derivative $dG*I(x)/dx$; (d) second derivative $d^2G*I(x)/d^2x$. The edge point in the original image brightness distribution (a) corresponds to a zero-crossing point in (d); the original image brightness distribution (a) intersects with its average level at the edge point.

$$\partial_t u = D_u \nabla^2 u , \quad (1)$$

$$\partial_t v = D_v \nabla^2 v , \quad (2)$$

where $u(x,y,t)$ and $v(x,y,t)$ are spatio-temporal variables in the two-dimensional spatial domain (x,y) and in the temporal domain t; ∂_t denotes $\partial/\partial t$ and $\nabla^2=\partial^2/\partial^2 x+\partial^2/\partial^2 y$ denotes the Laplacian operator. D_u and D_v are diffusion coefficients, which need to satisfy the relation $D_u<D_v$. We compute the time-evolving Eqs. (1) and (2), and then find zero-crossings in $(u-v)$ after a finite duration of time. The zero-crossings correspond to edge points.

Reaction-diffusion equations

The FitzHugh-Nagumo reaction-diffusion equations consist of the following two equations having two variables u and v [7,8]:

$$\partial_t u = D_u \nabla^2 u + [u(u-a)(1-u) - v]/\varepsilon , \quad (3)$$

$$\partial_t v = D_v \nabla^2 v + u - bv , \quad (4)$$

where a and b are constants and ε is a small constant. Let us consider the simple case of $D_u=D_v=0$ in the set of Eqs. (3) and (4), that is, the set of ordinary differential equations, as follows:

$$\mathrm{d}_t u = [u(u-a)(1-u) - v]/\varepsilon , \quad (5)$$

$$\mathrm{d}_t v = u - bv , \quad (6)$$

The system consisting of the ordinary differential Eqs. (5) and (6) becomes the mono-stable system having one stable equilibrium point or the bi-stable system having two stable equilibrium points. Figure 2 shows the system behaviour of the mono-stable system: a phase plot and temporal developments of u and v. The set of Eqs. (5) and (6) works as the time-dependent threshold function for the initial condition of u under the condition of v=0; the parameter a is the threshold value. In the mono-stable system, any solution (u,v) converges to the stable point (u,v)=(0,0).

The reaction-diffusion Eqs. (3) and (4) describe the system of non-linear oscillators coupled with diffusion; Eqs. (5) and (6) describes each of the non-linear oscillators. In the full reaction-diffusion Eqs. (3) and (4), when an initial phase difference between spatial neighbouring points is across the point $(u,v)=(a,0)$ of the phase plot, the system causes an impulse. Kuhnert et al. mentioned that the point of the impulse corresponds to an edge point [4]. However, the impulse does not remain at the point, but unfortunately travels in space according to the nature of the reaction-diffusion system [9].

The authors previously found that the discrete version of the set of Eqs. (3) and (4) detects edge points as a static state under the condition $D_u<D_v$, which is similar to the Turing condition [10]. Figure 3 shows an example of edge detection with the FitzHugh-Nagumo reaction-diffusion equations. The equations with a fixed value a divide image brightness into the two levels such as $u<a$ and $u>a$. Thus, impulses organized at the spatial positions having initial phase differences across the point $(u,v)=(a,0)$ do not necessarily correspond to edge points generally defined for grey level image.

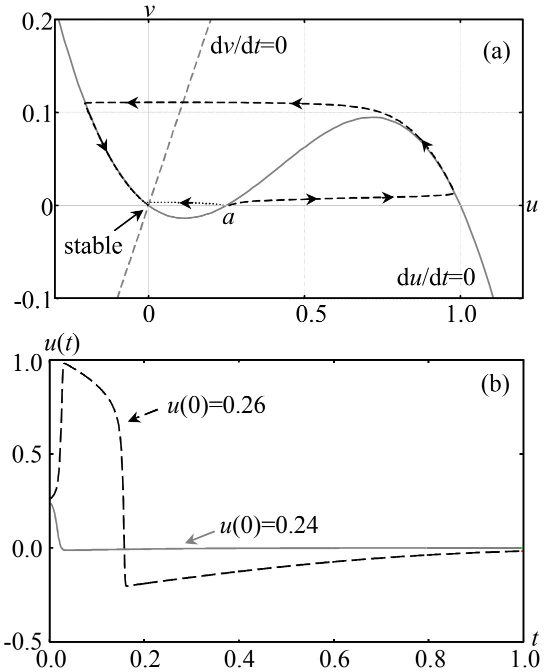

Fig. 2. System behaviour of the FitzHugh-Nagumo ordinary differential Eqs. (5) and (6). (a) Null-clines $u/dt=u(1-u)(u-a)-v=0$ and $dv/dt=u-bv=0$. A set of solutions (u,v) traces the trajectory indicated by the arrows. The origin $(u,v)=(0,0)$ is the stable equilibrium point. (b) Temporal developments of the solutions $u(t)$ having the two different initial conditions of $(u,v)=(0.26,0)$ and $(u,v)=(0.24,0)$. The parameter values utilized here were $a=0.25$, $b=1.0$ and $\varepsilon=1.0\times10^{-3}$.

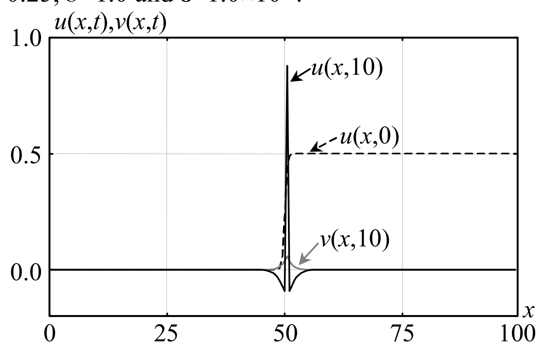

Fig. 3. Example of edge detection with the discrete version of the FitzHugh-Nagumo reaction-diffusion Eqs. (3) and (4) in one-dimensional space x. The broken line illustrates the initial condition of $u(x,0)$; the initial condition of $v(x,0)$ is zero for all over the space x. At $t=10$, the spatial distributions of $u(x,10)$ and $v(x,10)$ were obtained. Parameter values utilized here were $a=0.25$, $b=1.0$, $\varepsilon=1.0\times10^{-3}$, $D_u=1.0$ and $D_v=5.0$; the spatial finite difference was $\delta x=0.5$ and the temporal one was $\delta t=10^{-3}$.

Proposed algorithm

We propose an edge detection algorithm applicable to grey level image. As mentioned above, the FitzHugh-Nagumo reaction-diffusion equations detect edges for binarized image; the parameter a refers to the threshold value for binarisation. Thus, the parameter a needs to be modulated according to the local average level of the original grey level image. We couple the reaction-diffusion Eqs. (3) and (4) with the following simple diffusion equation having its diffusion coefficient D_a such as:

$$\partial_t a = D_a \nabla^2 a\,, \tag{7}$$

where $a(x,y,t)$ is a spatio-temporal variable, as opposed to the constant parameter in the original reaction-diffusion equations. The two diffusion coefficients D_u and D_v need to satisfy $D_u<<D_v$ for static edge patterns; the rest one D_a needs to be enough large for the computation of the local average level. Initial conditions for the three variables are

$$u(x,y,0) = a(x,y,0) = a_0 I(x,y)\,, \tag{8}$$

$$v(x,y,t=0) = 0\,, \tag{9}$$

where an image brightness distribution function $I(x,y)$ is normalized; the parameter a_0 is a constant and fixed at $a_0=0.25$.

For computing the partial differential equations (3), (4) and (7) numerically, we discretized the equations by the finite difference method with the Crank-Nicolson scheme; δx, δy and δt denote finite differences in space and time. The Neumann boundary condition for the three variables u, v and a governs the four sides of the rectangular domain of image. We solve an obtained set of linear equations by the successive over-relaxation method.

Experimental results and discussions

We applied the proposed edge detection algo-

Table 1. Parameter values utilized in experiments. Finite differences for discretisation in space and time are $\delta x = \delta y = 0.5$ and $\delta t = 10^{-3}$ for all of the three algorithms.

Algorithm and equations	Parameter values
DOG: Eqs. (1) and (2)	D_u=1.0, D_v=5.0
Original FitzHugh-Nagumo: Eqs. (3) and (4)	D_u=1.0, D_v=5.0 a=0.25, b=1.0, $\varepsilon = 10^{-3}$
Proposed algorithm: Eqs. (3), (4) and (7)	D_u=1.0, D_v=5.0, $D_a = 10^3$ a_0=0.25, b=1.0, $\varepsilon = 10^{-3}$

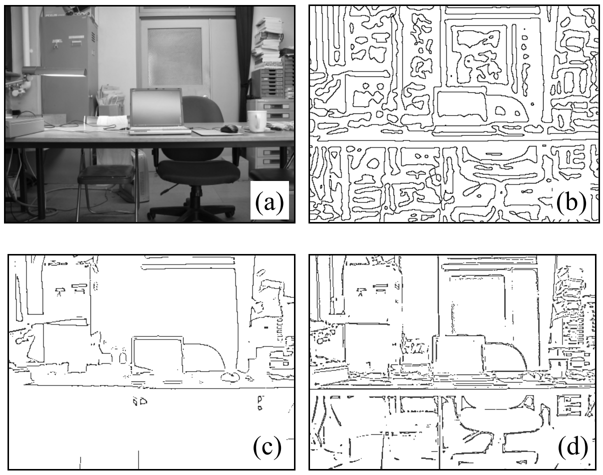

Fig. 4. Edge detection for an indoor scene. (a) Original image. Image size is 454×340 pixels with 256 brightness levels. Results by (b) DOG at t=1.0, (c) original FitzHugh-Nagumo at t=10 and (d) proposed algorithm at t=10. See Table 1 for parameter values utilized here.

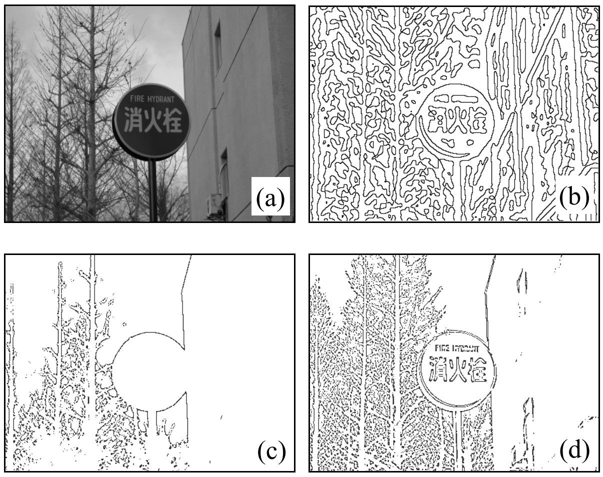

Fig. 5. Edge detection for an outdoor scene. (a) Original image. Image size is 454×340 pixels with 256 brightness levels. Results by (b) DOG at t=1.0, (c) original FitzHugh-Nagumo at t=10 and (d) proposed algorithm at t=10. See Table 1 for parameter values utilized here.

rithm and previous two algorithms to two real images. Table 1 summarizes the algorithms and their parameter values utilized in the experiments. Figures 4 and 5 show their edge detection results. The previous algorithm utilizing the difference of two Gaussian (DOG) filters detected edges at almost all points including weak gradients. The number of edge points detected by the previous algorithm utilizing the original FitzHugh-Nagumo reaction-diffusion equations is quite few. This is because the previous algorithm detects only edges of binarized image. The proposed algorithm provided better results in comparison with those of the previous algorithms.

Conclusion

The present paper proposed an edge detection algorithm utilizing the FitzHugh-Nagumo reaction-diffusion equations coupled with a simple diffusion equation. A local average level provided by the diffusion equation modulates the threshold level of the reaction-diffusion equations. The proposed algorithm works better than the previous ones; this was confirmed through experimental results obtained for real images.

References

1. D. Marr, E. Hildreth. Theory of Edge Detection. Proc. R. Soc. London, Ser. B, Biol. Sci., 1980. Vol. 207, P. 187-217
2. T. Sunayama, M. Ikebe, T. Asai, Y. Amemiya. Cellular νMOS Circuits Performing Edge Detection with Difference-of-Gaussian Filters. Jpn. J. App. Phys., 2000, Vol. 39, P. 2278-2286.
3. P. Perona, J. Malik. Scale-Space and Edge Detection Using Anisotropic Diffusion. IEEE Trans. Patt. Anal. Mach. Intell., 1990, Vol. 12, P. 629-639.
4. L. Kuhnert, K. I. Agladze, V. I. Krinsky. Image Processing Using Light-Sensitive Chemical Waves. Nature, 1989, Vol. 337, P. 244-247.
5. P. Mrázek, M. Navara. Selection of Optimal Stopping Time for Nonlinear Diffusion Filtering. Int. J. Comp. Vis., Vol. 52, 2003, P. 189-203.
6. A. Nomura, M. Ichikawa, H. Miike, M. Ebihara, H. Mahara, T. Sakurai. Realizing Visual Functions with the Reaction-Diffusion Mechanism. J. Phys. Soc. Jpn., 2003, Vol. 72, P. 2385-2395.
7. R. FitzHugh. Impulses and Physiological States in Theoretical Models of Nerve Membrane. Biophys. J., 1961, Vol. 1, P. 445-466.
8. J. Nagumo, S. Arimoto, S. Yoshizawa. An Active Pulse Transmission Line Simulating Nerve Axon. Proc. I. R. E., 1962, Vol. 50, P. 2061-2070.
9. J. D. Murray, Mathematical Biology, Springer-Verlag, Berlin, 1989.
10. A. M. Turing. The Chemical Basis of Morphogenesis. Philos. Trans. R. Soc. London, Ser. B, Biol. Sci., 1952, Vol. 237, P. 37-72.